	Indian Institute of Information Technology **Design and Manufacturing, Kancheepuram** Chennai 600 127, India	**COM-205T** `Discrete Structures for Computing` Quiz 1 26-Aug-2015 Duration: 1 hr Marks: 15

Name: **Roll no:**

0. `(0 marks) Prove that` $3^0 = 1$.

1. (1 mark) Show that $P \to (Q \to R) \leftrightarrow (P \to Q) \to (P \to R)$

2. (2 marks) Express the following using logic. P and Q are propositions. You are allowed to use only the following logical connectives: $\neg, \vee, \wedge, \to$ and any other operator is not allowed.

   - P is necessary for Q

   - P unless Q

   - P is necessary for Q whereas P is not sufficient for Q.

   - Either P or Q.

3. (1 mark) Negate the following: $\forall x \ \exists \epsilon((x > 0 \wedge \epsilon > 0) \wedge \forall y(y > 0 \to x - y \geq \epsilon))$

4. (1.5 marks) Consider the assertion: Discrete Mathematics grading is transparent only if I study Discrete Mathematics. For the given assertion, write the

   - Converse:

   - Inverse:

   - Contrapositive:

5. (0.5 marks) Write the following logical expression using only $\wedge$ and $\neg$. $P \to (Q \to P)$.

6. (2 marks) Consider the following two implications. Of the two, one is true and the other one is false. Justify your answer with a proof/counter example.
   (i) $\exists x(P(x) \wedge Q(x)) \to \exists x P(x) \wedge \exists x Q(x)$
   (ii) $\exists x P(x) \wedge \exists x Q(x) \to \exists x(P(x) \wedge Q(x))$

7. (1 mark) Check the validity of the argument.
Some trigonometric functions are periodic. Some periodic functions are continuous. Therefore, some trigonometric functions are continuous.

- Write the above argument using predicate logic.

- Prove or Disprove.

8. (2 marks) Prove or Disprove.

$$[\exists x P(x) \rightarrow \forall x Q(x)] \rightarrow \forall x[P(x) \rightarrow Q(x)]$$
$$\forall x[P(x) \rightarrow Q(x)] \rightarrow [\exists x P(x) \rightarrow \forall x Q(x)]$$

9. (1 mark) Express $\exists! x P(x)$ using $\forall x P(x)$ and $\exists x P(x)$. Your expression must involve both $\forall$ and $\exists$ and logically equivalent to $\exists! x P(x)$. Any other assumption must be stated clearly.

10. (1.5 marks) Express the following using First Order Logic. Clearly, mention UOD and the set of predicates used.
Some Republicans like all Democrats.
No Republican likes any Socialist.
Therefore, no Democrat is a Socialist.

11. (1.5 marks)`Scenario:` Five persons A, B, C, D, E are in a compartment in a train. A, C, E are men and B, D are women. The train passes through a tunnel and when it emerges, it is found that E is murdered. An inquiry is held, A, B, C, D make the following statements. Each makes two statements.

A says: I am innocent. B was talking to E when the train was passing through the tunnel.
B says: I am innocent. I was not talking to E when the train was passing through the tunnel.
C says: I am innocent. D committed the murder.
D says: I am innocent. One of the men committed the murder.

Out of 8 statements given above, 4 are true and 4 are false. Who is the murderer. Support your answer with a precise and concise justification.

Extra Credit: Prove or Disprove. All scientists are human beings. Therefore, all children of scientists are children of human beings.

ROUGH WORK

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India	**COM-205T** `Discrete Structures for Computing` Quiz 1 05-Oct-2015 Duration: 1 hr Marks: 15

Name: **Roll no:**

`0. (0 marks) What is your source (class notes, text books, internet) of preparation for COM 205T`

1. (1.5 mark) How many transitive relations are there on a set of size two. List all of them.

2. (1.5 marks) Consider the set $A = \{1, 2, \phi, \{1, 2\}\}$. Say true or false for the following.
   - $1, 2 \in A$
   - $\{1, 2\} \subset A$
   - $\{1, 2\} \in A$
   - $\phi \in A$
   - $\phi \subset A$
   - $\{\phi\} \subset A$

3. (1.5 marks) Consider the set of integers and the binary relation $R = \{(a, b) \mid \text{ a divides b}\}$. Is R an equivalence relation. Justify your answer with a proof/counter example.

4. (1 mark) How many binary relations are there on a finite set of size n that are symmetric and asymmetric. Justify.

5. (1 mark) `Claim:` If a binary relation R is symmetric and transitive, then R is an equivalence relation. `Proof:` Since R is symmetric, both (a, b) and (b, a) are in R and given that R is transitive, it follows that $(a, a) \in R$. Therefore, R is reflexive. From the above arguments, it follows that R is an equivalence relation. Is the proof correct. Justify your answer.

6. (1 mark) Prove or Disprove: $R_1R_2 \cap R_1R_3 \subset R_1(R_2 \cap R_3)$, where $R_1 \subseteq A \times B, R_2, R_3 \subseteq B \times C$.

7. (1.5 marks) Is it true that in a group of 5 people there exist 3 mutual friends or a pair of enemies (2 mutual enemies).

8. (1.5 marks) Prove or Disprove: in any set of 8 distinct integers there exist two whose sum or difference is divisible by 7.

9. (1.5 marks) Present a Direct Proof: $\forall n, 2^n \leq n! \leq n^n$

10. (1.5 marks) Present a proof using mathematical induction: $\forall n, 2^n \leq n! \leq n^n$

11. (1.5 marks) What is wrong with this `'proof'`. **Theorem:** For every positive integer n, if x and y are positive integers with $max(x, y) = n$, then $x = y$.

    Basis Step: Suppose that $n = 1$. If $max(x, y) = 1$ and x and y are positive integers, we have $x = 1$ and $y = 1$.
    Inductive Step: Let k be a positive integer. Assume that whenever $max(x, y) = k$ and x and y are positive integers, then $x = y$. Now let $max(x, y) = k + 1$, where x and y are positive integers. Then $max(x - 1, y - 1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step.

Extra Credit: How many equivalence (binary) relations are there on a set of size n. Justify.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India	**COM-205T** Discrete Structures for Computing End Semester 02-Dec-2015 Duration: 3 hrs Marks: 50

Name: **Roll no:**

0. The name of the Movie that narrates the discoveries of Prof.Nash Williams

1 Light Dose

Credits: 1 mark each

1. Write the power set of $\{\emptyset, \{\emptyset\}, \{1, 2\}\}$

2. Statement: A graph G is 2-colorable is a necessary condition for G to be bipartite. Write the converse and contrapositive.

3. Write the definition of Weak induction and Strong induction using the first order logic.

4. Let R_1, R_2 be relations defined on a finite set A and $t(R_1)$ is the transitive closure of R_1. Prove or Disprove. $t(R_1 \cup R_2) = t(R_1) \cup t(R_2)$

5. Given a function $f : A \to B$, what is the necessary and sufficient condition for f^{-1} to exist (inverse of f).

6. Let $A = \{1, \ldots, n\}$. Given a function $f : A \to A$ is onto, does it follow that f is 1-1. Prove or Disprove.

7. How many onto functions are there from a set of size 3 to a set of size 2.

8. How many binary strings are there of length 20 with exact 4 zeros.

9. Show that the greatest lower bound is unique.

10. A bag contains 3 red, 4 black, 5 blue balls. The minimum number of balls to be taken in any draw so that we get to see 3 balls of the same color.

11. Show that the set of composite numbers is infinite.

12. A= set of C-programs. B= set of C++ programs. Which set is bigger. Justify.

13. Draw a graph on 5 vertices such that G and $\bar{G}$ (complement of G) are same.

14. The maximum number of edges in a simple graph with 8 vertices and 4 components. Draw one such graph.

15. Is the number of graphs on n vertices with chromatic number 3 finite or infinite. Justify. Note: n is a fixed integer.

2 Medium Dose

`Credits: 1.5 marks each`

1. Express the following using the first order logic by clearly mentioning UOD, predicates used: Everyone who gets admitted into an IIT gets a job. Therefore, if there are no jobs, then nobody gets admitted into any IIT.

2. Suppose S and T are two sets and $f : S \to T$ is a function. Let R_1 be an equivalence relation on T. Let R_2 be a binary relation on S such that $(x, y) \in R_2$ iff $(f(x), f(y)) \in R_1$. Is R_2 an equivalence relation. Prove or Disprove.

3. Suppose R_1 and R_2 are equivalence relations (defined on a finite set A) inducing partitions $P1$ and $P2$. Let $R = R_1 \cap R_2$. How do you obtain the partition P induced by R using $P1$ and $P2$.

4. `Claim:` All students in DM course get 'S' grade. We now present a proof using mathematical induction on the number of students. `Base:` $n = 1$. 'Renjith' gets 'S' grade. `Hypothesis:` Assume $n = k$ students get 'S' grade. `Induction Step:` Consider a set of $k + 1$ students. The set $\{s_1, \ldots, s_{k+1}\}$ of students contain $\{s_1, \ldots, s_k\}$ and $\{s_2, \ldots, s_{k+1}\}$. Clearly both the sets are of size k and by the hypothesis all students in $\{s_1, \ldots, s_k\}$ get 'S' grade and all students in $\{s_2, \ldots, s_{k+1}\}$ get 'S' grade. Therefore, all students in $\{s_1, \ldots, s_{k+1}\}$ get 'S' grade. This completes the induction. Is the proof correct. If not, identify the flaw in this argument.

5. A= set of prime numbers and the binary relation R is 'divides' ; Is R a partial order. Is R a well-order. What are the minimal elements of the set $\{2, 3, 5, 7\}$. What are the minimal elements of the set A.

6. Let A be a finite set and R be a binary relation on A. Count the following sets.

   - The number of irreflexive and symmetric binary relations

   - The number of irreflexive and anti-symmetric binary relations

   - The number of irreflexive and asymmetric binary relations

7. Show that one of any n consecutive integers is divisible by n.

8. Show that the number of derangements on n items is $\sum_{i=2}^{n}(-1)^{i-1}\frac{n!}{i!}$.

9. Show that the set $[3, 4]$ is uncountable.

10. Prove that G is bipartite if and only if G is 2-colorable. Be precise and formal.

3 Strong Dose

`Credits: 2 marks each`

1. All horses are animals. Therefore, heads of horses are heads of animals. Prove or Disprove.

2. How many partial orders are there on a set of size 3. List all of them.

- How of them are total order.

- How many of them are well-order.

3. Given \$4 and \$5 currency, is it possible give change for \$$n$ using these denominations. If yes, prove using Mathematical Induction.

4. Two disks, one smaller than the other , are each divided into 200 congruent sectors. In the larger disk 100 of the sectors are chosen arbitrarily and painted red; the other 100 sectors are painted blue. In the smaller disk each sector is painted either red or blue with no stipulation on the number of red and blue sectors. The small disk is then placed on the larger disk so that their centers coincide. Show that it is possible to align the two disks so that the number of sectors of small disk whose color matches the corresponding sector of the large disk is atleast 100. (Hint: PHP)

5. An infinite integer array is passed as an input to a sorting program. How many different inputs are possible, i.e., is it finite or countably infinite or uncountable. Justify.

6. Draw two non-isomorphic graphs with degree sequence $(3, 3, 3, 3, 3, 3, 3, 3, 3, 3)$ if it exists. Intuitively, argue that the two graphs drawn are non-isomorphic. Justify if no such graphs exist for this degree sequence.

7. With suitable justifications, find the cardinality of the following sets (finite, countably infinite, uncountable)

   - The number of acyclic graphs on n-vertices, n is a fixed integer.

   - The number of bipartite graphs on n-vertices, $n \in \mathbf{N}$. Note: n is a variable.

8. Mention a set and a relation satisfying the following conditions
    - a subset with no maximal element and no minimal element.
    - a subset with no lub and no glb

9. Seven students go on holidays. They decide that each will send a post card to three of the others. Is it possible that every student receives post cards from precisely the three to whom he sent postcards.

10. How many binary equivalence relations are there on a set of size n. Prove your answer. Be precise.

SPACE FOR ROUGH WORK

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 205T - Discrete Mathematics	Quiz 1 30-Aug-2016 Duration: 1hr Marks: 15

Roll No: Name:

0. (0 marks) Name the scientist with whom mathematician Ramanujam had a good academic career

1. (1 mark) I prepare well for exams is sufficient for me to get good grades. And, I secure good grades only if I maintain a good CGPA.

2. (1 mark) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists x P(x) \wedge \exists x Q(x)$. Let us attempt a proof.
   By definition; $(P(0) \wedge Q(0)) \vee (P(1) \wedge Q(1)) \vee (P(2) \wedge Q(2)) \vee \ldots$
   What is the next step ? Complete the proof. Do not attempt any other proof technique.

3. (1 mark) Negate the following and simplify.
$\forall n \exists z \forall k(|z| = k \to \exists u \exists v \exists w((z = uvw \land |uv| \geq k \land |v| \geq 1) \land \forall i(i \geq 0 \to uv^iw \in S)))$

4. (1 mark) Prove or Disprove: $\forall x(P(x) \leftrightarrow Q(x)) \leftrightarrow \exists x(P(x) \leftrightarrow Q(x))$

5. (1 mark) What is the underlying meaning of the following logical expression; P is some predicate.
$\exists x(P(x) \land \forall y(P(y) \leftrightarrow y = x))$

6. (3 marks) Write logical notation for each of the following; for each, write an expression using only existential quantifier and an another expression using only universal quantifier. UOD: Set of students. PREDICATES: $Boy(x)$ x is a boy, $SMART(x)$ x is smart. Do NOT use any other predicates.

   (a) Some boys are smart.

   Using only $\exists$

   Using only $\forall$

   (b) Not all boys are smart.

   Using only $\exists$

   Using only $\forall$

   (c) All boys are not smart.

   Using only $\exists$

   Using only $\forall$

7. (2 marks) Some students of DM are well motivated by a faculty. All students of DM likes all faculty. Therefore, some students of DM likes a faculty who motivates them. UOD: Set of students and faculty, PREDICATES: $STUD(x)$: x is a student. $FACULTY(x)$: x is a faculty. $LIKES(x, y)$: x likes y. $MOTIVATES(x, y)$: x motivates y.

   - Write the above argument in FOL.

- Is the above argument true ?

8. (2 marks) There exists a IIIT where many students are studying. There is a IIIT with no students. Therefore, there are two IIITs such that a student is part of one IIIT whereas he is not part of the other. UOD: Set of students and IIITs. PREDICATES: $STUD(x)$: x is a student. $IIIT(x)$: x is a IIIT. $STUDY(x, y)$: x is studying in y. Do NOT use any other predicates.

   - Write the above argument in FOL.

   - Is the above argument true ?

9. (3 marks) Consider the academic timetable at IIITDM. UOD: Set of students,courses and time slots. PREDICATES: $STUD(x)$: x is a student. $ELECOURSE(x)$: x is an elective course. $COURSE(x)$: x is a course. $TIMESLOT(x)$: x is a time slot. $TAKEN(x,y)$: x has taken course y. $DAY(x,y)$: (course) x is offered on (day) y. $COURSE\text{-}OFFERED\text{-}SLOT(x,y)$: x is offered in time slot y. Write the FOL for the following.

- Each student has taken at least two elective courses.

- There exists a student who has courses in all time slots. (there exists a student who has taken at least one course in each time slot)

- There is a student who has not taken a course on any of the time slots on Wednesday.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India	**COM-205T** Discrete Structures for Computing End Semester 02-Dec-2015 Duration: 3 hrs Marks: 50

Name: **Roll no:**

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9. Show that the set $[3, 4]$ is uncountable.

10. Prove that G is bipartite if and only if G is 2-colorable. Be precise and formal.

3 Strong Dose

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- How of them are total order.

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7. With suitable justifications, find the cardinality of the following sets (finite, countably infinite, uncountable)

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   - The number of bipartite graphs on n-vertices, $n \in \mathbf{N}$. Note: n is a variable.

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   - a subset with no maximal element and no minimal element.
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9. Seven students go on holidays. They decide that each will send a post card to three of the others. Is it possible that every student receives post cards from precisely the three to whom he sent postcards.

10. How many binary equivalence relations are there on a set of size n. Prove your answer. Be precise.

SPACE FOR ROUGH WORK

	Indian Institute of Information Technology **Design and Manufacturing, Kancheepuram** Chennai 600 127, India	**COM-205T** `Discrete Structures for Computing` Quiz 1 26-Aug-2015 Duration: 1 hr Marks: 15

Name: **Roll no:**

`0. (0 marks) Prove that` $3^0 = 1$.

1. (1 mark) Show that $P \rightarrow (Q \rightarrow R) \leftrightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$

2. (2 marks) Express the following using logic. P and Q are propositions. You are allowed to use only the following logical connectives: $\neg, \vee, \wedge, \rightarrow$ and any other operator is not allowed.

   - P is necessary for Q
   - P unless Q
   - P is necessary for Q whereas P is not sufficient for Q.
   - Either P or Q.

3. (1 mark) Negate the following: $\forall x \ \exists \epsilon ((x > 0 \wedge \epsilon > 0) \wedge \forall y (y > 0 \rightarrow x - y \geq \epsilon))$

4. (1.5 marks) Consider the assertion: Discrete Mathematics grading is transparent only if I study Discrete Mathematics. For the given assertion, write the

   - Converse:

   - Inverse:

   - Contrapositive:

5. (0.5 marks) Write the following logical expression using only $\wedge$ and $\neg$. $P \rightarrow (Q \rightarrow P)$.

6. (2 marks) Consider the following two implications. Of the two, one is true and the other one is false. Justify your answer with a proof/counter example.
   (i) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists x P(x) \wedge \exists x Q(x)$
   (ii) $\exists x P(x) \wedge \exists x Q(x) \rightarrow \exists x(P(x) \wedge Q(x))$

7. (1 mark) Check the validity of the argument.
Some trigonometric functions are periodic. Some periodic functions are continuous. Therefore, some trigonometric functions are continuous.

- Write the above argument using predicate logic.

- Prove or Disprove.

8. (2 marks) Prove or Disprove.

$$[\exists x P(x) \to \forall x Q(x)] \to \forall x[P(x) \to Q(x)]$$
$$\forall x[P(x) \to Q(x)] \to [\exists x P(x) \to \forall x Q(x)]$$

9. (1 mark) Express $\exists! x P(x)$ using $\forall x P(x)$ and $\exists x P(x)$. Your expression must involve both $\forall$ and $\exists$ and logically equivalent to $\exists! x P(x)$. Any other assumption must be stated clearly.

10. (1.5 marks) Express the following using First Order Logic. Clearly, mention UOD and the set of predicates used.
Some Republicans like all Democrats.
No Republican likes any Socialist.
Therefore, no Democrat is a Socialist.

11. (1.5 marks)`Scenario:` Five persons A, B, C, D, E are in a compartment in a train. A, C, E are men and B, D are women. The train passes through a tunnel and when it emerges, it is found that E is murdered. An inquiry is held, A, B, C, D make the following statements. Each makes two statements.

A says: I am innocent. B was talking to E when the train was passing through the tunnel.
B says: I am innocent. I was not talking to E when the train was passing through the tunnel.
C says: I am innocent. D committed the murder.
D says: I am innocent. One of the men committed the murder.

Out of 8 statements given above, 4 are true and 4 are false. Who is the murderer. Support your answer with a precise and concise justification.

Extra Credit: Prove or Disprove. All scientists are human beings. Therefore, all children of scientists are children of human beings.

ROUGH WORK

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM - Discrete Structures for Computing	Quiz 2 05-Oct-2017 Duration: 1hr Marks: 15

Roll No: Name:

0. (0 marks) How many books did the mathematician Ramanujan read during his research career.

1. (0.5 marks) What is the power set of $\{1, 2, \{1, 2\}\}$.

2. (1 mark) Say true or false with exactly one line justification.
   (i) $\{1, 2\} \in \{1, 2, \{1, 2\}\}$

   (ii) $\{1, 2\} \subset \{1, 2, \{1, 2\}\}$

3. (1.5 marks) Let $A = \{1, 2, 3\}$ and R be a binary relation defined on A. Present an example relation R such that
   (i) R is symmetric and anti symmetric

   (ii) R is anti symmetric but not reflexive

   (iii) R is neither symmetric nor transitive

4. (1.5 mark) Five distinct non-negative numbers are chosen randomly from the set of integers. Prove or disprove: there exists two in the chosen set such that their sum or difference is divisible by 6.

5. (1 mark) 21 numbers are chosen randomly from the set of integers. What is the tight lower bound on the set of integers that are divisible by 3 in the chosen set. Justify.

6. (2 marks) Prove or disprove: For every integer k, there are more than $k+3$ prime numbers.

7. (2 marks) Show that $\sqrt{5}$ is irrational.

8. (2 marks) Claim: in any group of 13 people, there exists 4 mutual friends or 3 mutual enemies. Present a proof or a counter example.

9. (1.5 marks) Write the base cases and the inductive hypothesis for the following claim. Do NOT prove this claim. A monkey is asked to climb a ladder of size n (n steps). Each time, it takes either 1 step or 2 steps or 3 steps. Claim: The number of ways of climbing up the ladder is at most 4^n.

10. (2 marks) Let A be a set. Like binary, ternary relations, are there unary relations defined on A. What are they and how many are there. Prove your answer.

Extra credit: (2.5 marks) What is the minimum number of people in a group so that we either find 4 mutual enemies or 4 mutual friends. Prove your answer.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 205T - Discrete Mathematics	End Semester 17-Nov-2017 Duration: 3hrs Marks: 40

Roll No: Name:

0. Name the scientist who discovered the theory of infinite sets

1 Light Dose

1 mark each

1. Write converse and inverse for the statement 'I drink coffee whenever I get headache'.
   Converse:

   Inverse:

2. Out of the following the four logical expressions, identify the two that are equivalent. NO justification is needed.
   (a) $\forall x(P(x) \rightarrow Q(x))$
   (b) $\neg\exists x(P(x) \wedge \neg Q(x))$
   (c) $\neg\neg\exists x(P(x) \vee \neg Q(x))$
   (d) $\forall x(P(x) \vee \neg Q(x))$

3. Is there a binary relation which is both reflexive and irreflexive. Mention one, if exists.

4. How many 1-1 functions are there from a domain of size 4 to co-domain of size 3.

5. Prove or Disprove: In a group of 5 people there exists 3 mutual friends or 3 mutual enemies.

6. Draw a graph for the degree sequence $(3, 3, 3, 1, 1, 1)$.

7. Consider the above graph drawn as a binary relation, and find the transitive closure.

8. How many onto functions are there from a domain of size 4 to a co-domain of size 2.

9. What is the chromatic number of a complete graph on $n \geq 2$ vertices.

10. Draw a planar graph for the degree sequence $(4, 4, 4, 4, 1, 1, 1, 1)$, if it exists.

2 Medium Dose

`2 marks each`

1. Is the Peterson graph an Eulerian graph. How about the Line graph of the Peterson graph. Justify.

2. Show that for any planar graph, $V - E + F = 2$.

3. Show that $[5, 9]$ is uncountable.

4. On a set of size n, how many binary equivalence relations are there ? Prove your answer.

5. Present an example set and a subset for each of the following
   - Minimum and Maximum elements exist, however neither least nor greatest elements exist
   - Upper and lower bounds exist, however neither greatest LB nor least UB exist
6. How many binary relations are there that are neither reflexive nor antisymmetric.

7. The set A consists of composite numbers and the set B consists of prime numbers. Which set is larger. Justify.

8. In how many different ways can k pigeons be distributed into n pigeonholes such that each pigeon has at least two pigeons.

9. $\forall x(P(x) \rightarrow Q(x)) \rightarrow \forall x P(x) \vee \forall x Q(x)$. Is this true ? How about the converse ?

3 Strong Dose

3 marks each

1. Draw example graphs satisfying (i) Both G and $\bar{G}$ (complement of G) are planar (ii) G is planar whereas $\bar{G}$ is non-planar (iii) Both G and $\bar{G}$ are non-planar

2. Present a **good** lower and upper bound for non-transitive binary relations. Justify.

3. Express the following in FOL: Some logicians are good at proof techniques. Not all logicians are good at graph theory, however all logicians are good at some topics in infinite sets. Therefore, there are logicians who are neither good at functions nor relations.

4. Consider the series 1 2 2 4 8 32 256 .... What is the n^{th} number in this series. Present a good upper bound and a proof of correctness if deriving exact number is challenging.

Extra Credit: (3 marks) Express in FOL. All horses are animals. Therefore, heads of horses are heads of animals. Prove or disprove.

Indian Institute of Information Technology
Design and Manufacturing, Kancheepuram
Chennai 600 127, India
An Autonomous Institute under MHRD, Govt of India
An Institute of National Importance
COM 205T - Discrete Mathematics

Quiz 1
04-Sep-2018
Duration: 1hr
Marks: 15

Roll No: Name:

1. (3 marks) Write the following in logic using logical notation.
   (a) P unless Q
   (b) P is sufficient for Q but not necessary for Q
   (c) It is not the case that P only if Q
2. (1 mark) I shall attend DM or skip DSA. Write the negation of this statement.
3. (3 marks) I attend DM lecture if it is interesting. Write the
   (a) Converse
   (b) Inverse
   (c) Contrapositive
4. (2 marks) Prove or Disprove the following logical arguments; All students of DM like DSA. Some students of DM like Design. Therefore, some students of DM do not like Design.

5. (2 marks) Write the definition of prime number in FOL. Clearly mention UOD and predicates used.

6. (2 marks) Write FOL.

   (a) Some objects do not satisfy $P(x)$.

   (b) Not all objects satisfy $P(x)$.

   (c) None of the objects satisfy $P(x)$.

   (d) Any object satisfy $P(x)$.

7. (2 marks) Prove or Disprove: $\forall x(P(x) \rightarrow Q(x)) \rightarrow \forall x P(x) \rightarrow \forall x Q(x)$

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COM 205T - Discrete Mathematics

Quiz 2
08-Oct-2018
Duration: 1hr
Marks: 15

Roll No: Name:

1. (1 mark) Write the power set of $\{\emptyset, \{1, 2\}, 3\}$.

2. (1 mark) Let A be a finite set and $R \subseteq A \times A$. What is the least value of A (minimum number of elements) such that R is symmetric but not antisymmetric.

3. (1 mark) $A = \{a, b, c\}$. List all unary relations of A.

4. (1 mark) Prove or disprove; If a relation is symmetric and asymmetric then it is anti-symmetric.

5. (1 mark) Let A be a set of size n. How many binary relations (defined on A) are there that are not reflexive. Justify your answer.

6. (1.5 marks) Let $A = \{1, 2, 3, 4\}$. $R = \{(1, 1), (1, 2), (2, 3)\}$. Find reflexive, symmetric and transitive closure of R.

7. (2 marks) Prove using Mathematical Induction; clearly mention the base case, hypothesis and the induction step. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} < 1$.

8. (1.5 marks) Is the following proof correct. Claim: All CED students secure grade 'S' in DM course. Proof is by induction on n, the number of students. `Base:` $n = 1$. Vaibhav is awarded grade 'S'. `Hypothesis:` Assume the claim is true for a class of $k \geq 1$ students. That is in a class of k students, all get grade 'S'. `Induction step:` Consider a class of size $k+1, k \geq 1$. Let $S = S_1, S_2, \ldots, S_{k+1}$ denote the set of students. Clearly, S can be seen as $\{S_1, \ldots, S_k\} \cup \{S_2, \ldots, S_{k+1}\}$ and by the induction hypothesis, the claim is true in the above two sets. Therefore, all students in any $k+1$ size class get grade 'S'. Thus, the claim follows.

9. (1.5 marks) Prove that the set of natural numbers is infinite. Hint: Proof by contradiction.

10. (2 marks) Prove or Disprove: Given a set A and $R_1, R_2 \subseteq A \times A$ such that R_1 and R_2 are equivalence relations. `Claim:` $R_1 \cap R_2$ is an equivalence relation.

11. (1.5 marks) Prove using MI: $x^0 = 1$ for any integer x.

`Extra Credits:`

(i) (2 marks) Count the number of binary relations that are neither reflexive nor antisymmetric.

(ii) (3 marks) Show that $T_n \geq n! + B_n - 1$ where T_n is the number of transitive relations and B_n is the number of partitions of a set of size n.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 205T - Discrete Mathematics	End Sem 19-Nov-2018 Duration: 3hr Marks: 40

Roll No: Name:

1 Light Dose

1 mark each

1. Draw two different simple graphs with the degree sequence $(2, 2, 2, 2, 2, 2)$

2. Verify Euler's Planarity formula for the above two graphs.

3. Draw two different graphs with the degree sequence $(3, 3, 3, 3, 3, 3)$ such that the first graph contains a triangle whereas the second graph does not.

4. Write the statement ' All lions are animals' using (i) universal quantifier only (ii) existential quantifier only.

5. Prove or Disprove: The intersection of two infinite sets is infinite.

6. Let $A = \{1, 2, 3, 4, 5\}$, $R = \{(1, 1), (2, 2), (1, 3), (4, 5)\}$. Find (i) Reflexive closure (ii) Symmetric closure

7. How many onto functions are there from a set of size 4 to to a set of size 3.

8. Let $R = \{(a, b) \mid a, b \in \mathbf{R}$ and a divides b $\}$. Is R an equivalence relation.

9. Let R_1 and R_2 are partial order relations defined on a finite set. Prove or disprove; $R_1 \cap R_2$ a partial order.

10. How many binary relations are irreflexive and asymmetric.

2 Medium Dose

1.5 marks each

1. Write Euclid's division lemma in FOL; any positive integer a can be divided by any other positive integer b in such a way that it leaves a remainder r that is smaller than b. Clearly define UOD and the predicates used.

2. Is it true that the number of non-transitive binary relations is at least $2^{\frac{n^2-n}{2}} - 1$. Justify.

3. Draw two different graphs with the degree sequence $(3, 3, 3, 3, 3, 3, 3, 3, 3, 3)$ such that one has Hamiltonian cycle whereas the other does not.

4. Show that countable union of countable sets is countable.

5. Show that $\sqrt{3}$ is irrational.

6. Let $A = \{1, 2, \ldots, n\}$. What is the binary relation R defined on A such that (i) R has maximum number of distinct equivalence classes (ii) R has least number of distinct equivalence classes

7. Suppose we have stamps of two denominations 4 cents and 7 cents. We want to know is it possible to make up exactly any postage of 18 cents or more using these denominations. Prove the claim using MI.

8. Prove that the number of equivalence relations (Bell's number) is upper bounded by 2^{n^2}. You may use any proof technique of your choice.

9. Draw Hasse Diagrams satisfying (i) maximal elements but no greatest element (ii) upper bounds but no least upper bound. Clearly, mention the set, subset and relation considered for discussion.

10. Prove: $\exists x(P(x) \vee Q(x)) \leftrightarrow \exists x P(x) \vee \exists x Q(x)$.

3 Strong Dose

2.5 marks each

1. Draw a 4-colorable graph with no triangles. Justify that the chromatic number of the graph drawn is 4.

2. What should be the value of n (lower bound for n) such that in any group of n people, there exists either 4 mutual friends or 4 mutual enemies. Justify your answer.

3. How many different graphs are there on n vertices, if (i) n is a fixed integer. (ii) n is a variable integer. Justify.

4. Show that the decimal expansion of a rational number is either terminating or if it is non-terminating then it is repeating. Hint: Pigeon hole principle.

5. Write the following in FOL; UOD: set of sets. A set is finite if and only if it is not infinite. It is not the case that all sets are infinite. Some infinite sets are either countable or uncountable, but not both. There are at least two infinite sets whose cardinalities are same. For each infinite set it is always the case that all its elements are of finite length. For each infinite set, not all its subsets are infinite. Therefore, there exists an infinite set with some of its elements are of infinite length is false.

6. Given a set of size n, how many binary relations are antisymmetric. Prove your answer using the principle of mathematical induction. Clearly mention, the base case, hypothesis and the induction step.

`Extra Credits:`

(i) (2 marks) How many C-programs are there having exactly one printf and scanf statements. Justify.

(ii) (3 marks) $A = [0, 1]$ (the closed interval, real line 0 to 1), $B = \mathbf{R}$. (the set of real numbers). Which set is bigger. Justify your answer.

Outstanding next

36.5

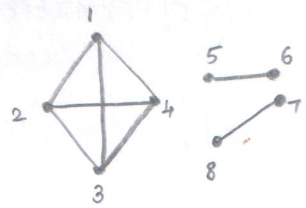
	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 205T Discrete Mathematics	End Semester 24-Nov-2016 Duration: 3hr Marks: 40

Roll No: COE15B003 Name: S. PRANAVE

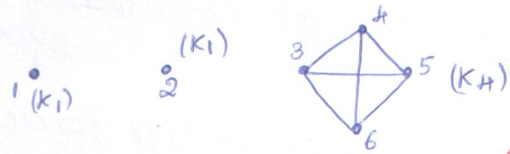
1 Light Dose

1 mark each

1. Draw a graph with the degree sequence $(3, 3, 3, 3, 1, 1, 1, 1)$.



2. Draw a graph on 6 vertices having 3 components with maximum number of edges.



max no. of edges = 6.

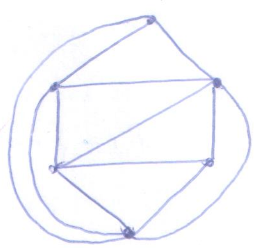
3. Show that $K_6 - 3e$ is planar.

~~If G is planar, then $|e| \le 3n-6$.~~

~~$K_6 - 3e$ has $\frac{6 \times 5}{2} - 3 = 12$~~

~~$3n-6 = 3 \times 6 - 6 = 12$~~

$K_6 - 3e$ is planar



4. Verify Euler's planarity formula for Trees.

If G is planar, then $|E| \le 3n-6$

Tree is connected acyclic graph, $|E| = n-1$

$\therefore n-1 \le 3n-6$.

5. How many onto functions are there from a set of size 3 to a set of size 2.

From $m=3$, $n=2$

$= n^m - n_{c_1}(n-1)^m + (n-2)^m \times n_{c_2}$

$= 2^3 - 2_{c_1}(1)^3 + (0)^3\, 2_{c_2}$

$= 8 - 2 = 6$

6. How many Derangements are there on the set $\{1, 2, 3, 4\}$

$n = 4$

$n! - {}^nC_1 (n-1)! + {}^nC_2 (n-2)! - {}^nC_3 (n-3)! + {}^nC_4 (n-4)!$

$= 4! - 4 \times 3! + 4C_2 \times 2! - 4C_3 \times 1! + 4C_4 \times 0! = 9$

7. Count the number of irrational numbers. Clearly mention your assumptions.

- We know that number of real numbers is uncountable and number of rational numbers is countably infinite

Countable union of countable sets is countable.

Assume no. of irrational numbers is countable. Countable union of rational and irrational numbers is countable. which is a contradiction

∴ Our assumption is wrong. ∴ No. of Irrational numbers is uncountable.

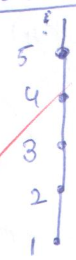
8. List all subsets of $A = \{\{1, 2\}, 1, 2\}$.

$\{1\}, \{2\}, \{\{1,2\}\}, \{1,2\}, \{1,\{1,2\}\}, \{2,\{1,2\}\}, \{\}, \{\{1,2\},1,2\}$

Wellorder total order + any subset of I^+ should have least element.

9. Is $(I^+, \leq)$ a well order. Justify.

$I^+ = \{1, 2, 3, 4, 5, \dots\}$



Yes, $(I^+, \leq)$ is a well order.

In chain like structure with lower bound for the set is well order.

10. Let $A = \{1, 2, 3, 4\}$. List all relations which satisfy both equivalence and partial order properties.

$R_1 = \{(1,1), (2,2), (3,3), (4,4)\}$

that is reflexive, symmetric & antisymmetric.

11. Express using FOL. Some boys are slow in reading than all boys but at least one boy in class reads faster than every boy

$B(x)$: x is boy

$Slow(x)$: x is slow in reading than y

$\neg Slow(x)$: x is fast in reading than y.

$\exists x [B(x) \wedge \forall y [B(y) \longrightarrow Slow(x,y)]]$

$\wedge \; \exists z [B(z) \wedge \forall y [B(y) \longrightarrow \neg Slow(z,y)]]$

12. Express using FOL. There is a barber who shaves all men in the town who do not shave themselves

$B(x)$: x is a barber

Shave(x, y) : x shaves y

Man(y) : y is the man in the town

$$\exists x \left[B(x) \wedge \forall y \left[\left(\text{Man}(y) \wedge \neg \text{shave}(y,y) \right) \rightarrow \text{shave}(x,y) \right] \right]$$

13. Express using FOL. A student in this class has not read the book and everyone in this class cleared DM course. Therefore, someone who cleared DM has not read the book.

$$\exists x \, [\text{Stud}(x) \wedge \neg \text{Read}(x)]$$

$$\wedge$$

$$\forall x \, [\text{Stud}(x) \rightarrow DM(x)]$$

$$\rightarrow \exists x \, [\text{Stud}(x) \wedge DM(x) \wedge \neg \text{Read}(x)]$$

14. Is the above claim true. Justify.

$$(\exists x \, [S(x) \wedge \neg R(x)]) \wedge (\forall x \, (S(x) \rightarrow DM(x))) \rightarrow \exists x \, (S(x) \wedge DM(x) \wedge \neg R(x))$$

By EI, $S(c) \wedge \neg R(c)$ — ①

① ⇒ $S(c)$, for some c — ②

① ⇒ $\neg R(c)$ " — ③

By UI, $S(c) \rightarrow DM(c)$ — ④ for any arbitrary c

From ② & ④, $DM(c)$ — ⑤

From ②, ③ and 5,

$DM(c) \wedge \neg R(c) \wedge S(c)$

By EG, $\exists x \, [DM(x) \wedge \neg R(x) \wedge S(x)]$

15. Show that $\forall x(P(x) \vee Q(x)) \rightarrow \forall x P(x) \vee \exists x Q(x)$.

contrapositive:

$$\neg(\forall x \, p(x) \vee \exists x \, Q(x)) \rightarrow \neg \forall x \, (p(x) \vee Q(x))$$

$$\neg \forall x \, p(x) \wedge \neg \exists x \, Q(x) \rightarrow \exists x \, \neg (p(x) \vee q(x))$$

$$\exists x(\neg p(x)) \wedge \forall x (\neg Q(x)) \rightarrow \exists x \, (\neg p(x) \wedge \neg Q(x))$$

$$\exists x \, R(x) \wedge \forall x \, S(x) \rightarrow \exists x (R(x) \wedge S(x))$$

$$\rightarrow \exists x \, R(x) \wedge \exists x \, S(x)$$

$\because \{ \exists x \, (p(x) \wedge q(x)) \rightarrow \exists x \, p(x) \wedge \exists x \, q(x)$

a: $\exists x R(x)$	b: $\forall x S(x)$	c: $\exists x S(x)$	$a \wedge b$	$a \wedge c$	
T	T	T	T	T	T
T	T	F	X	X	X
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	F	F	T
F	T	F	X	X	X
F	F	T	F	F	T
F	F	F	F	F	T

Excellent

$[\because (p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

$\neg P(x) = R(x)$

$\neg Q(x) = S(x)$ $]$

Since the last column is Tautology,

$\forall x \, (P(x) \vee Q(x)) \rightarrow \forall x \, p(x) \vee \exists x \, Q(x)$

2 Medium Dose

1.5 marks each

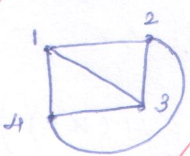
1. Show that in any graph there exists two vertices receiving the same degree.

Case 1: Vertices can have degree from $\{0, 1, \dots n-2\}$ ← pigeonholes and n pigeons (vertices). By PHP, there exist a pigeonhole with same ~~number of~~ degree ∴ There exists two vertices having same degree

Case 2: Vertices can have degree from $\{1, 2, \dots n-1\}$ ← pigeonholes and n pigeons (vertices). By PHP, there exist a pigeonhole with 2 pigeons that is there exists two vertices receiving same degree.

2. Draw a 4-colorable planar graph. Present a plane drawing and justify that the graph is not 3-colorable.

K_4 is 4-colorable planar graph.



Let c_1 be the color of V_1

Then, ∵ V_2, ~~V_3~~, ~~V_4~~ is adjacent to V_1,

let c_2 be the color of V_2. $(c_2 \neq c_1)$

Since V_3 is adjacent to both V_1 and V_2

let c_3 be the color of V_3. $(c_3 \neq c_1$ & $c_3 \neq c_2)$

Now V_4 is adjacent to V_1, V_2, V_3 and let c_4 be the color of V_4 $(c_4 \neq c_1$ & $c_4 \neq c_2$, & $c_4 \neq c_3)$ ∴ 4 differenct colors are required to K_4.

Hence, not 3-colorable.

3. Show that G is 3-partite iff G is 3-colorable.

If G is 3-colorable, then G is 3-partite.

G is 3-partite ⇒ $V_1 \cap V_2 \cap V_3 = \phi$

$V_1 \cup V_2 \cup V_3 = V$

and V_1, V_2, V_3 are independent sets.

∴ $V_1 = \{ u \mid \chi(u) \text{ is red} \}$

$V_2 = \{ v \mid \chi(v) \text{ is green} \}$

$V_3 = \{ w \mid \chi(w) \text{ is blue} \}$

If G is 3-partite, then G is 3-colorable,

Since G is 3-partite, $u, v \in V_i$, then $(u, v) \notin E(G)$, $1 \leq i \leq 3$

∴ Vertices in V_1 can be colored using c_1

Vertices in V_2 can be colored using c_2

Vertices in V_3 can be colored using c_3.

4. How many cycles are there in a graph on n-vertices. Justify whether the count is finite/countably infinite/uncountable.

If n is fixed, then $^nC_3 + {^nC_4} + {^nC_5} + \cdots {^nC_n}$

are the number of cycles in a graph with n-vertices.

$\therefore$ count is finite

1

5. Count the number of integer matrices of order $m \times n$, where m and n are fixed integers. **Note:** Entries of the matrix come from $\mathbb{I}$. Justify whether the count is finite/countably infinite/uncountable.

I is countably infinite.

Counting the number of integer matrices is equivalent to the cardinality of $I \times I \times I \times \cdots$ mn times

Since m and n are fixed integers, mn is also fixed.

$\therefore$ Cardinality of $I \times I \times I \times \cdots$ mn times is countably infinite.

$\therefore$ The number of integer matrices of order $m \times n$ are countably infinite if m and n are fixed.

1

6. Show that the power set of Σ^* is uncountable, $\Sigma = \{0, 1, 2\}$.

We know that Σ^* is countably infinite.

Assume $P(\Sigma^*)$ is countable $\Rightarrow \exists$ a enumeration.

Let $A_1, A_2, A_3, \ldots$ be subset containing $x_1, x_2, x_3, \ldots \in \Sigma^*$

	x_1	x_2	x_3	x_4
A_1	0	1	1	0 ...
A_2	1	0	0	0 ...
A_3	1	1	1	0
⋮				

1

To show that enumeration is incomplete,

Let $B = \{ x_i \mid (A_i, x_i) = 0 \}$, $B \in P(\Sigma^*)$.

But B is not listed in enumeration

$\therefore$ which contradicts our assumption.

$\therefore P(\Sigma^*)$ is uncountable.

7. How many equivalence relations are there on a set of size 6. Present a precise bound.

Number of equivalence reln's on a set of size 6 = No. of partitions = Bell's number

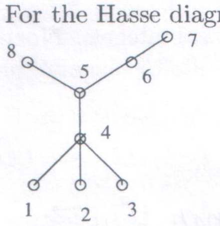
$B_0 = 1, B_1 = 1, B_2 = 2, B_3 = 5, B_4 = 15, B_5 = 52.$

$B_6 = \sum_{k=0}^{5} B_{5-k}\, {}^5C_k = B_5\, {}^5C_0 + B_4 \times {}^5C_1 + B_3 \times {}^5C_2 + B_2 \times {}^5C_3 + B_1 \times {}^5C_4 + B_0 \times {}^5C_5$

$= 52 \times 1 + 15 \times 5 + 5 \times \frac{5 \times 4}{2} + 2 \times 10 + 1 \times 5 + 1 \times 1$

$= 203$

8. For the Hasse diagram given below;



- Find maximal elements for $\{1, 2, 3, 5\}$ $\{5\}$
- Find upper bounds for $\{3, 5\}$. Also, find lub. $\{5, 8, 6, 7\}$

  Lub $= \{5\}$
- Find lower bounds for $\{4\}$. $\{4, 1, 2, 3\}$

9. Coin exchange: Show that for any $n \geq n_0$, the change for n can be given using denominations Re 7 and Re 5. Prove using M.I.

B.C: For $n = 24$, denominations 2 (Re. 7) and 2 (Re. 5)

Indn Hypothesis: For $n = K$, $K \geq 24$. Change for K can be given using Re 7 & Re 5.

Indn step: For $n = K+1$,

case ①: 2 (7 rupees) can be replaced with 3 (5 rupees)

$K+1 = K - 2(7) + 3(5)$

case ②: If there are no. 7 rupees, there atleast 5 (5 rupees) are required to make up denomination (24).

4 (5 rupee) coins can be replaced with 3 (7 rupee coins)

$\therefore K+1 = K - 4(5) + 3(7)$

$\therefore$ The change for $n \geq 24$, can be given using denominations Re 7 and Re 5

$\therefore$ Hence proved.

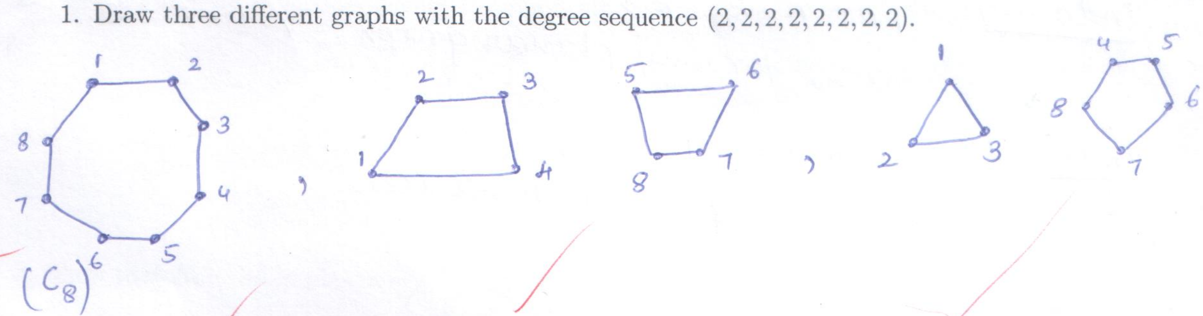
10. Show that the decimal expansion of a rational number, must after some point terminates (for ex: $\frac{4}{5}$) or becomes periodic (the same sequence starts repeating, e.g., $\frac{1}{3}$). Prove using PHP.

The digit after the decimal point lies in the Subset $\{1, \ldots 9\}$
Assume if zero comes after the decimal point, then expansion terminates and there are ~~atleast~~ more than 9 digits after decimal point. According to PHP, there are more than 9 pigeons and 9 pigeonholes, implies there exist a pigeon holes with more than one pigeon. Meaning a digit ~~is se~~ has occured second time in the decimal expansion. ∴ becomes periodic.
∴ Decimal expansion of a rational number, must after some point terminates or becomes periodic.

3 Strong Dose

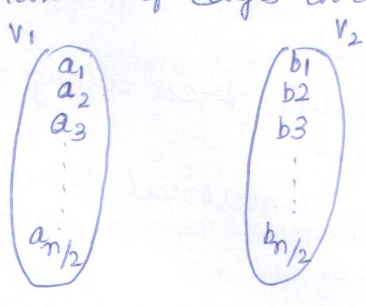
2 marks each

1. Draw three different graphs with the degree sequence (2, 2, 2, 2, 2, 2, 2, 2).



2. What is the minimum and maximum number of edges in a bipartite graph on n vertices. Assume n is even.

Max no. of edges in a bipartite graph, V_1 (contains $n/2$ vertices)
V_2 (")



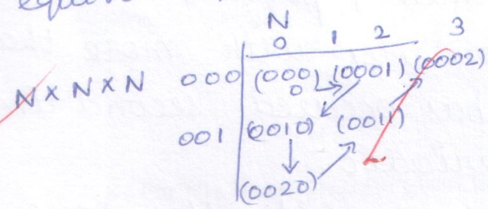
a_1 is connected ~~from~~ to $(b_1, b_2, \ldots b_{n/2})$
a_2 is connected to $(b_1, b_2, \ldots b_{n/2})$
⋮
$a_{n/2}$ is connected to $(b_1, b_2, \ldots b_{n/2})$
∴ $n/2 \times n/2 = n^2/4$ (max no. of edges)

Minimum number of edges $= n/2$ [V_1 $(n-1)$ vertices) / V_2 (1 vertex)]

3. Let $A = \{1, 2, 3, 4\}$ and $B = \mathbb{N}$. How many functions exist between A and B. Justify whether the count is finite/countably infinite/uncountable.

Since 1 can be mapped to any one of the natural numbers, there are $\mathbb{N}$ possibilities for 1, ||ly for 2, 3, and 4.

is equivalent to the cardinality of $N \times N \times N \times N$ ∴ There exists enumeration



∴ cardinality of $N \times N \times N \times N$ = Countably infinite

∴ No. of function is countably infinite.

4. Show using mathematical induction that for any planar graph, $V - E + F = 2$.

Base case: For $n = 3$, $3 - 3 + 2 = 2$

∴ Base case is satisfied.

Indn Hypo: For $n = k$, $k \geq 3$ for any planar graph, $V - E + F = 2$.

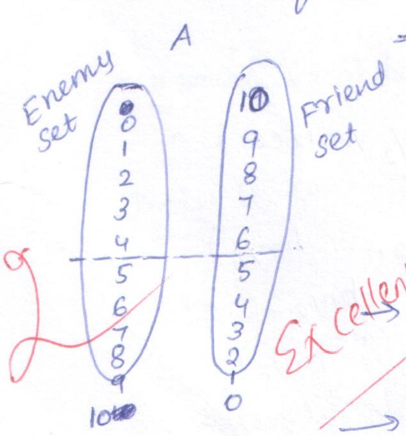
Indn Step: For $n = k+1$, $k \geq 3$

Let G be graph with n vertices,

5. Show that in any group of 11 people, there exists 3 mutual enemies or 4 mutual friends.

consider the friend set and enemy set are divided w.r.to A.

⇒ A has atleast 6 friends or atleast 5 enemies.



case ①: atleast 6 friends.

In a group of 6 people, there exists 3 mutual enemies or 3 mutual friends.

Excellent

→ If there are 3 mutual friends, then along with A, there are 4 mutual friends

→ otherwise there exists 3 mutual enemies.

case 2: atleast 5 enemies.

→ If there are atleast 2 enemies among 5 people, then along with A, there exists 3 mutual enemies.

→ Otherwise, all 5 are friends ⇒ 5 mutual friends.

Indian Institute of Information Technology
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An Institute of National Importance
COM 205T - Discrete Mathematics

Quiz 2
06-Oct-2016
Duration: 1hr
Marks: 15

Roll No: Name:

0. (0 marks) What is your source of preparation for COM 205.
(i) No preparation (ii) Class notes only (iii) class notes + text books (iv) Others

1. (1 mark) Let $A = \{1, 2\}$ and $R \subseteq A \times A$. List all binary relations R that are reflexive and symmetric.

$R_1 = \{(1,1)\ (2,2)\}$

$R_2 = \{(1,1)\ (2,2)\ (1,2)\ (2,1)\}$

2. (1 mark) Let $|A| = n$ and $R \subseteq A \times A$. How many binary relations are there that are irreflexive and antisymmetric. Justify your answer.

R_i # irr + Antisy $= 3^{\frac{n^2-n}{2}}$

0/1 (1,2) (1,3) . . .
0/1 (2,1) (3,1)
Include one of them or None — $\frac{n^2-n}{2}$
3 possi

3. (1 mark) Are there binary relations that are reflexive and irreflexive ? Justify.

If $A = \phi$ $R = \phi$ $\Rightarrow$ Ref + irr

$A \neq \phi$ No such rel^n exists

4. (1 mark) Let $A = \{1, 2, 3, 4, 6, 7, 8, 9\}$. Show that if we pick any subset A' containing 5 elements from A, then there exists a pair in A' such that their sum or difference is divisible by 10.

(1,9) (2,8) (3,7) (4,6)

5 pigeons 4 boxes

By php, atleast one box contains ≥ 2 pigeons.

5. (1 mark) Let $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (b, c), (c, a), (d, e)\}$. Find the transitive closure of R.

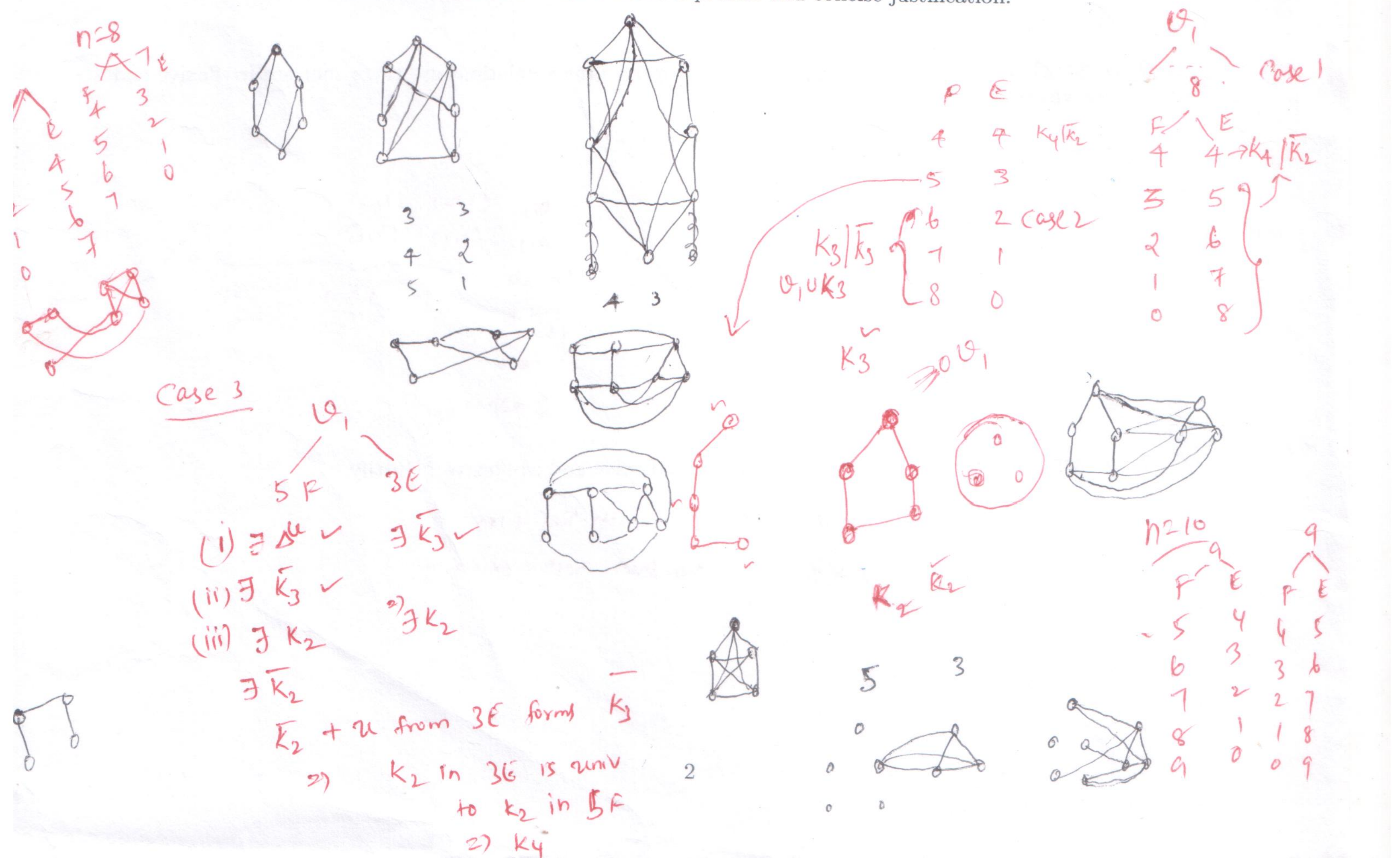
$R_1 = \{ (a,b) (b,c) (c,a) (d,e) \}$

$R_2 = \{ (a,c) (b,a), (c,b) \}$.

$R_3 = \{ (a,a) (b,b) (c,c) \}$

$R_1 \cup R_2 \cup R_3$.

6. (3 marks) What is the value of n (minimum n) such that in any group of n people you see either 3 mutual enemies or 4 mutual friends. Present a precise and concise justification.



7. (2 marks) Given n pigeons to be distributed among k pigeonholes:
What is a necessary and sufficient condition on n and k that, in every distribution, at least two pigeonholes must contain the same number of pigeons. Justify your answer.

$0 \quad 1 \quad \cdots \quad k-2$ ⊔ $\leq k-2$

$\frac{(k-1)(k-2)}{2} + \leq k-2$

$n \leq \frac{k-1(k-1)}{2} + k-2$

$(k-2)\left[\frac{k-1}{2}+1\right]$

$\frac{(k-2)}{2}\left[k-1+2\right]$

$\frac{(k+1)(k-2)}{2}$

8. (2.5 marks) Given two denominations *Re* 3 and *Re* 5, show using mathematical induction that for all $n \geq n_0$, exact change for n can be given using these two denominations.

Base:

Hypothesis:

Induction Step:

9. (2.5 marks) Let $|A| = n$ and $R \subseteq A \times A$. How many binary relations R satisfy antisymmetric property. Prove your answer using mathematical induction.

Base:

Hypothesis:

Induction Step:

$\exists x\,(P(x) \to Q(x))$

$\to \exists x\,P(x) \to \exists x\,Q(x).$

$P \to Q$

Light 1, 2, 6 – 5
Medium 4, 7, 8 – 5
Strong 3, 5, 9 – 5
15

Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 205T - Discrete Mathematics	Quiz 1 30-Aug-2016 Duration: 1hr Marks: 15

Roll No:

Name:

0. (0 marks) Name the scientist with whom mathematician Ramanujam had a good academic career

1. (1 mark) I prepare well for exams is sufficient for me to get good grades. And, I secure good grades only if I maintain a good CGPA.

1 min

$(P \to Q) \wedge (Q \to R)$

$P, P \to Q \to Q.$

2. (1 mark) $\exists x(P(x) \wedge Q(x)) \to \exists xP(x) \wedge \exists xQ(x)$. Let us attempt a proof.
By definition; $(P(0) \wedge Q(0)) \vee (P(1) \wedge Q(1)) \vee (P(2) \wedge Q(2)) \vee \ldots$
What is the next step ? Complete the proof. Do not attempt any other proof technique.

2 min

$(P(0) \wedge Q(0)) \vee (P(1) \wedge Q(1)) \vee (P(2) \wedge Q(2))$

$[(P(0) \wedge Q(0)) \vee P(1)] \wedge [(P(0) \wedge Q(0)) \vee Q(1)]$

$(P(0) \vee P(1)) \wedge (Q(0) \vee P(1)) \wedge (P(0) \vee Q(1)) \wedge (Q(0) \vee Q(1))$

Ignore $P \wedge Q \to P$.

$(P(0) \vee P(1)) \wedge (Q(0) \vee Q(1))$

$\exists x\,P(x) \wedge \exists x\,Q(x).$

5 min

3. (1 mark) Negate the following and simplify.
$\forall n \exists z \forall k(|z| = k \rightarrow \exists u \exists v \exists w((z = uvw \wedge |uv| \geq k \wedge |v| \geq 1) \wedge \forall i(i \geq 0 \rightarrow uv^i w \in S)))$

$\exists n \forall z \exists k \, (|z| = k \wedge \forall u \forall v \forall w \, ((z = uvw \wedge |uv| \geq k \wedge |v| \geq 1) \rightarrow \exists i \, (i \geq 0 \wedge uv^i w \notin S)))$

1 min

4. (1 mark) Prove or Disprove: $\forall x(P(x) \leftrightarrow Q(x)) \leftrightarrow \exists x(P(x) \leftrightarrow Q(x))$

$\rightarrow$ ✓ $\quad \forall x \, (P(x) \leftrightarrow Q(x))$

UI $P(a) \leftrightarrow Q(a) \rightarrow$ for some a $P(a) \leftrightarrow Q(a)$

$\rightarrow \exists x \, (P(x) \leftrightarrow Q(x))$

$\leftarrow$ X $\quad P(x) : x = 2 \quad Q(x) : x$ is even

$\exists x \, (P(x) \leftrightarrow Q(x)) \not\rightarrow \forall x \, (P(x) \leftrightarrow Q(x))$

T F False.

3 min

5. (1 mark) What is the underlying meaning of the following logical expression; P is some predicate.
$\exists x(P(x) \wedge \forall y(P(y) \leftrightarrow y = x))$

There exists unique x such that $P(x)$ $\quad \exists! x \, P(x)$

$\forall UOD\ \forall P\ \forall Q$ Claim. $\rightarrow$ Proof.

$\neg$ proof $\rightarrow$ $\exists UOD\ \exists P\ \exists Q\ \neg$Claim
(C. Ex)

6. (3 marks) Write logical notation for each of the following; for each, write an expression using only existential quantifier and an another expression using only universal quantifier. UOD: Set of students. PREDICATES: $Boy(x)$ x is a boy, $SMART(x)$ x is smart. Do NOT use any other predicates.

10 min

   (a) Some boys are smart.

   Using only ∃ $\exists x\,(Boy(x) \wedge Smart(x))$

   Using only ∀ $\neg \forall x\,(Boy(x) \rightarrow \neg Smart(x))$

   (b) Not all boys are smart.

   Using only ∃ $\exists x\,(Boy(x) \wedge \neg Smart(x))$

   Using only ∀ $\neg \forall x\,(Boy(x) \rightarrow Smart(x))$

   (c) All boys are not smart.

   Using only ∃ $\neg \exists x\,(Boy(x) \wedge Smart(x))$

   Using only ∀ $\forall x\,(Boy(x) \rightarrow \neg Smart(x))$

7. (2 marks) Some students of DM are well motivated by a faculty. All students of DM likes all faculty. Therefore, some students of DM likes a faculty who motivates them. UOD: Set of students and faculty, PREDICATES: $STUD(x)$: x is a student. $FACULTY(x)$: x is a faculty. $LIKES(x, y)$: x likes y. $MOTIVATES(x, y)$: x motivates y.

10 min

   - Write the above argument in FOL.

$\exists x\,(Stud(x) \wedge \exists y\,(faculty(y) \wedge Motivates(y, x)))$

$\forall x\,(Stud(x) \rightarrow \forall y\,(faculty(y) \rightarrow Likes(x, y)))$

$\exists x\,(Stud(x) \wedge \exists y\,(Faculty(y) \wedge Likes(x, y) \wedge Motivates(y, x)))$

$P(1)$

$\forall K\,(P(K) \rightarrow P(K+1))$

$\forall n\,P(n)$.

$\forall K\,(P(1) \wedge \cdots \wedge P(K) \rightarrow P(K+1))$

$\forall n\,P(n)$.

- Is the above argument true ?

$S(a) \wedge F(b) \wedge M(b,a)$ for some a, b.

$S(a) \rightarrow (F(b) \rightarrow L(a,b))$ for any a, b

$\therefore F(b) \rightarrow L(a,b)$ $[S(a) \wedge S(a) \rightarrow (F(b) \rightarrow L(a,b))]$

$L(a,b)$ $[F(b) \wedge F(b) \rightarrow L(a,b)]$

$L(a,b) \wedge M(b,a)$

Hence, the claim

8. (2 marks) There exists a IIIT where many students are studying. There is a IIIT with no students. Therefore, there are two IIITs such that a student is part of one IIIT whereas he is not part of the other. UOD: Set of students and IIITs. PREDICATES: $STUD(x)$: x is a student. $IIIT(x)$: x is a IIIT. $STUDY(x, y)$: x is studying in y. Do NOT use any other predicates.

10 min

- Write the above argument in FOL.

$\exists y (IIIT(y) \wedge \exists x (Stud(x) \wedge Study(x, y)))$

~~$\exists x \exists y (Stud(x) \wedge IIIT(y) \wedge Study(x,y))$~~

$\exists x (IIIT(x) \wedge \forall y (Stud(y) \rightarrow \neg Study(y, x)))$

$\exists y_1 \exists y_2 (IIIT(y_1) \wedge IIIT(y_2) \wedge \exists x (Stud(x) \wedge Study(x, y_1) \wedge \neg Study(x, y_2)))$, $y_1 \neq y_2$

(or) $\exists y_1 \exists y_2 (IIIT(y_1) \wedge IIIT(y_2) \wedge \exists x (Stud(x) \wedge ([Study(x,y_1) \wedge \neg St(x,y_2)] \vee [Study(x,y_2) \wedge \neg St(x,y_1)])))$, $y_1 \neq y_2$

or

$\neg (St(x,y_1) \leftrightarrow St(x,y_2))$

- Is the above argument true ?

$S(a) \wedge I(b) \wedge St(a,b)$

$I(c) \wedge (S(d) \rightarrow \neg St(c,d))$ True for 'a'

$\neg \exists y (stud(x, IIIT))$

$I(c) \wedge S(a) \wedge (S(a) \rightarrow \neg St(c,a))$

$\neg St(c,a)$

$I(b) \wedge I(c) \wedge S(a) \wedge St(a,b) \wedge \neg St(c,a)$

True.

9. (3 marks) Consider the academic timetable at IIITDM. UOD: Set of students,courses and time slots. PREDICATES: $STUD(x)$: x is a student. $ELECOURSE(x)$: x is an elective course. $COURSE(x)$: x is a course. $TIMESLOT(x)$: x is a time slot. $TAKEN(x,y)$: x has taken course y. $DAY(x,y)$: (course) x is offered on (day) y. $COURSE\text{-}OFFERED\text{-}SLOT(x,y)$: x is offered in time slot y. Write the FOL for the following.

15 min

- Each student has taken at least two elective courses.

$\forall x\,(Stud(x) \rightarrow \exists y_1, \exists y_2\,(y_1 \neq y_2 \wedge taken(x, y_1) \wedge taken(x, y_2) \wedge EleCourse(y_1) \wedge EleCourse(y_2)))$

$\forall x\,(Stud(x) \rightarrow \neg \exists! y\,(Electivecourse(y) \wedge taken(x, y)))$

(or) $\neg \exists x\,(Stud(x) \wedge \exists! y\,(E(y) \wedge taken(x, y)))$

- There exists a student who has courses in all time slots. (there exists a student who has taken at least one course in each time slot)

$\exists x\,(Stud(x) \wedge \forall y\,(timeslot(y) \rightarrow \exists z\,(Course(z) \wedge Course\_offered\_slot(z, y) \wedge taken(x, z))))$

- There is a student who has not taken a course on any of the time slots on Wednesday.

$\exists x\,(Stud(x) \wedge \forall y\,(time\,slot(y) \rightarrow \forall z\,(Course(z) \wedge Courseoffendslot(z, y) \wedge Day(z, WED) \rightarrow \neg taken(x, z))))$

or

$\exists x\,(Stud(x) \wedge \forall y\,(y = wednesday \rightarrow \forall z\,(timeslot(z) \rightarrow \neg \exists u\,((course(u) \wedge taken(x, u) \wedge Courseoffend(u, z))))$

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COM 205T - Discrete Mathematics

Quiz 1
23-Aug-2019
Duration: 1hr
Marks: 15

Roll No:

Name:

0. (0 marks) The documentary 'this film needs no title' is about the contribution of the scientist .....

1. (1.5 marks) DM course is interesting only if I participate in class room discussions. Write

Converse:

If I participate in classroom discussion then DM course is interesting

Inverse:

If DM course is not interesting then I didn't participate in classroom discussion

Contrapositive:

If I didn't participate in classroom discussion then DM course is not interesting

2. (1 mark) Write FOL using predicates $S(x)$: x is a student, $F(x)$: x is a faculty, $ST(x)$: x is a staff, $ID(x)$: x participated in Independence day celebrations and $RD(x)$: x participated in Republic day celebrations. Do not use any other predicate. Students, Faculty and Staffs of IIITDM participated in the Independence day and the Republic day celebrations.

$\forall x\ ((S(x) \vee F(x) \vee ST(x)) \rightarrow ID(x) \wedge RD(x))$

$\forall x \forall y \forall z\ (x \neq y \neq z\ \ S(x) \wedge F(y) \wedge ST(z) \rightarrow ID(x) \wedge ID(y) \wedge ID(z) \wedge RD(x) \wedge RD(y) \wedge RD(z))$

3. (1 mark) Write the principle of Mathematical Induction in FOL.

$[P(0) \wedge P(1) \wedge \forall n \geq 1\ (P(n) \rightarrow P(n+1))] \rightarrow \forall n \geq 0\ P(n)$

4. (3 marks) Write FOL. UOD: set of persons, $Like(x, y)$: x likes y

(a) Some one likes some one
$\exists x \exists y (like(x,y))$

(b) Some one likes all
$\exists x \forall y (like(x,y))$

(c) Each one likes every one
$\forall x \forall y\ (like(x,y))$

(d) No one likes every one
$\neg \exists x \forall y\ (like(x,y))$ // one likes everyone is false

(e) None likes all
$\neg \exists x \forall y (like(x,y))$ // There exist a person who likes all is false

(f) Each one likes no one
$\forall x \forall y (\neg like(x,y))$ // Each one likes someone is false

i) Each one likes everyone
$\forall x \forall y\ like(x,y)$

ii) Someone doesnot like some
$\exists x \exists y \neg like(x,y)$

5. (1.5 marks) Prove or Disprove: $[\forall x P(x) \rightarrow \exists x Q(x)] \leftrightarrow [\exists x (P(x) \rightarrow Q(x))]$

$\forall x P(x) \rightarrow \exists x Q(x)$

$\leftrightarrow \neg \forall x P(x) \vee \exists x Q(x)$

$\leftrightarrow \exists x \neg P(x) \vee \exists x Q(x)$

$\leftrightarrow \exists x (\neg P(x) \vee Q(x))$

$\leftrightarrow \exists x\ (P(x) \rightarrow Q(x))$

7) a) iii) $\exists x\,(IIIT(x) \wedge T(x) \wedge S(x) \wedge \forall y\,(IIT(y) \wedge T(y) \rightarrow learn(x,y)))$

6. (1.5 marks) Negate and Simplify: $\forall x \exists y (P(y) \wedge Q(x,y) \wedge \forall z (R(z) \rightarrow \exists w (S(z,w) \wedge T(x,y,z,w))))$

$\exists x \forall y ((P(y) \wedge Q(x,y)) \rightarrow \exists z (R(z) \wedge \forall w (S(z,w) \rightarrow \neg T(x,y,z,w))))$

7. (2.5 marks) Translate the following context into first order logical statements. Use only the defined predicates and none else. UOD: set of all people, $student(x)$: x is a student, $IIIT(x)$: x is part of IIIT, $IIT(x)$: x is part of IIT, $Teacher(x)$: x is a teacher, $AssociatedWith(x,t)$: x associated with institute t, $learns(x,y)$: x learns from y, $interact(x,y)$: x interacts with y.

(a) There is a teacher at IIIT who is a student himself learns from every teacher at IIT.

i) $\exists x\,(IIIT(x) \wedge T(x) \wedge \forall y\,(IIT(y) \wedge T(y) \rightarrow learn(x,y)) \wedge S(x))$

ii) $\exists x\,(IIIT(x) \wedge T(x) \wedge \exists z\,(IIT(z) \wedge S(z) \wedge x = z) \wedge \forall y\,(IIT(y) \wedge T(y) \rightarrow learn(x,y)))$

(b) Each teacher at IIIT has at least three students from whom they learn the subject and at most two teachers at IIT with whom they interact and learn the subject.

$\forall x\,[IIIT(x) \wedge T(x) \rightarrow \exists u \exists v \exists w\,(u \neq v \neq w \wedge learns(x,u) \wedge learns(x,v) \wedge learns(x,w))$

$\wedge \neg(\exists s \exists t \exists p\,(s \neq t \neq p \wedge T(s) \wedge T(t) \wedge T(p) \wedge (interact(x,s) \wedge interact(x,t) \wedge interact(x,p) \wedge learns(x,s) \wedge learns(x,t) \wedge learns(x,p))))]$

8. (1.5 marks) Consider a logical argument given in logical notation. Check the validity of the argument without using truth table. $[((A \vee B) \rightarrow C) \wedge (B \rightarrow (C \vee D)) \wedge ((\neg A \wedge \neg B) \rightarrow C) \wedge (\neg C \rightarrow B)] \rightarrow (\neg A \vee C \vee D)$. You are asked to check whether $(\neg A \vee C \vee D)$ follows from (inferred from) the given argument. Present clear justification for each statement inferred by you.

$A \vee B \rightarrow C$ is true from premise

$\leftrightarrow (\neg A \vee C) \wedge (\neg B \vee C)$ is TRUE

$\leftrightarrow \neg A \vee C$ is TRUE

$\leftrightarrow \neg A \vee C \vee D$ is TRUE

Conclusion follows

9. (1.5 marks) For the expression, $\forall x (P(x) \leftrightarrow Q(x))$, write three different equivalent expressions.

1) $\forall x ((P(x) \rightarrow Q(x)) \wedge (Q(x) \rightarrow P(x)))$

2) $\neg \exists x ((P(x) \wedge \neg Q(x)) \vee (Q(x) \wedge \neg P(x)))$

3) $\forall x ((\neg Q(x) \rightarrow \neg P(x)) \wedge (\neg P(x) \rightarrow \neg Q(x)))$

4) $\forall x\,(\neg(P(x) \oplus Q(x)))$

5) $\forall x ((\neg P(x) \wedge \neg Q(x)) \vee (Q(x) \wedge P(x)))$

(4 marks) Extra Credit: (Use additional sheet) Prove or Disprove: All professors are teachers. Therefore, all deans of professors are deans of teachers.

$\forall x\,(P(x) \rightarrow T(x))$

$\forall x (\exists x (P(y) \wedge Dean(x,y)) \rightarrow \exists y (T(y) \wedge Dean(x,y)))$

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COM 205T - Discrete Mathematics

Quiz 2
11-Oct-2019
Duration: 1hr
Marks: 15

Roll No:

Name:

0. (0 marks) What is your source of preparation (i) text book (ii) scribe (iii) lecture notes (iv) any other ....

1. (1 mark) List all subsets of the set $\{\phi, \{1,2\}, 1, 2\}$

$n = 4$
\# subsets $= 2^4 = 16$

$\{\phi\}, \{\{1,2\}\}, \{1\}, \{2\}, \{\phi, \{1,2\}\}, \{\phi, 1\}, \{\phi, 2\}, \{\{1,2\}, 1\}, \{\{1,2\}, 2\}, \{1,2\}$
$\{\phi, \{1,2\}, 1\}, \{\phi, \{1,2\}, 2\}, \{\phi, 1, 2\}, \{\{1,2\}, 1, 2\}, \{\phi, \{1,2\}, 1, 2\}, \phi$

2. (1 mark) Let $A = \{1,2,3\}$. Present an example relation which is

(i) reflexive and antisymmetric

$R = \{(a,b) \mid a, b \in A \wedge a \leq b\}$ $\quad R_1 = \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,3)\}$

(ii) asymmetric and transitive

$R_2 = \{(a,b) \mid a, b \in A \wedge a < b\}$ $\quad R_2 = \{(1,2), (1,3), (2,3)\}$

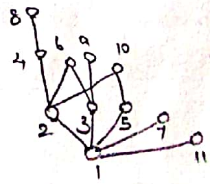
3. (1 mark) Let $A = \{1,2,3,4\}$ and $R = \{(1,2),(2,3),(3,4)\}$. Find the transitive closure and the asymmetric closure of R.

$t(R) = R \cup \{(1,3), (1,4), (2,4)\}$

$A(R) = R$

4. (1 mark) $A = \{1, \ldots, 11\}$. $R = \{(a,b) \mid a \text{ divides } b\}$. List

- Maximal elements of A $\quad \{8, 6, 9, 10, 7, 11\}$
- Minimal elements of $\{3,6\}$ $\quad \{3\}$
- Lower bounds of $\{3,6,11\}$ $\quad \{1\}$
- Upper bounds of $\{2,4\}$ $\quad \{4, 8\}$

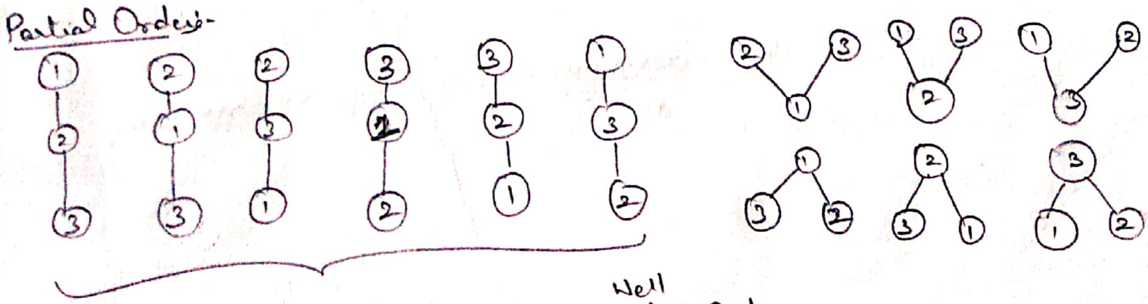


5. (1 mark) What is wrong with the following proof; **Claim:** For each positive integer n, $2^n < 3^n$. `Base:` $2 < 3$ is true. `Hypothesis:` Assume $2^n < 3^n, n \geq 1$. `Induction Step:` Consider $2^{n+1} < 3^{n+1}$. This can be rewritten as $2 \times 2^n < 3 \times 3^n$. By the base case, $2 < 3$ and by the hypothesis $2^n < 3^n$, therefore, $2 \times 2^n < 3 \times 3^n$ is true. Thus, the induction step is true. Therefore, the claim follows.

Induction Step Consider both LHS and RHS which is wrong. One should consider either LHS or RHS to solve or can consider both the sides of Induction Hypo for solving.

6. (1 mark) List all total orders on a set of size 3. How many of them are well orders.

Partial Order:-



Total Order as well as Well partial Order.

7. (2 marks) Given a set A and $R \subseteq A \times A$, we define the following property P " if (a, b) and (b, a) are in R then $a \neq b$ ". How many R satisfy this property P. Present a (i) direct proof (ii) proof by mathematical induction to justify your claim.

direct proof

We cannot choose diagonal elements and from remaining $n^2 - n$ elements, the number of binary relations $= 2^{n^2-n} \equiv$ # irreflexive relation

proof by MI

Base case: $A = \{1\}$ $R = \{\phi\}$. $|R| = 1 = 2^{1-1} = 1$

$A = \{1, 2\}$ $R = \{\phi\}, \{(1,2)\}, \{(2,1)\}, \{(1,2), (2,1)\}$ $|R| = 4$ $2^2 = 4$

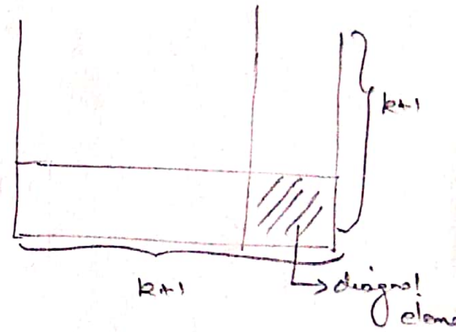
Hypo: Assume for a k sized set, # of such relation $= 2^{k^2-k}$, $\forall k \geq 2$ is true.

Induction Step: for a (k+1) sized set, we have to prove that $2^{(k+1)^2 - (k+1)}$

By Hypo, for a k sized set, we know that 2^{k^2-k}

for a $k+1^{th}$ element, new elements $= (k+1) + (k+1) - 2$ (since (k+1, k+1) is not allowed)

$= 2k$



relations on (k+1) sized set $= (2^{k^2-k}) \times 2^{2k}$

$= 2^{k^2+k}$

$= 2^{(k+1)^2 - (k+1)}$

$\therefore$ for a set of size n, no of rel^n $= 2^{n^2-n}$ is true.

Method 2:-

Diagonal elements should not be there in R. Therefore, $\exists$ 4 possibilities for an (a_i, a_j) pair ie (a_i, a_j) is present in R, (a_j, a_i) is present in R, (a_i, a_j) and (a_j, a_i) is in R or both are not present in R.

There are $(n^2-n)/2$ pairs of (a_i, a_j) is in R. # R $= 4^{\frac{n^2-n}{2}}$

8. (1.5 marks) CSE 18 batch has 121 students across 27 states. Two students are related if they belong to the same state. How many equivalence classes are there. Justify. Is it true that there exists a state with at least 4 students. Justify.

By generalized PHP, $\exists$ a state with atleast $\left\lceil \frac{121}{27} \right\rceil = 5$ students

27 Equivalence classes, Each state corresponds to one equivalence class

9. (2 marks) We define nice-transitive property of R as follows; if $(a, b), (b, c), (c, d) \in R$, then $(a, d) \in R$. A relation is a 'nice-equivalence relation' if it is reflexive, symmetric and nice-transitive. How many nice-equivalence relations are there on a set of size n. Prove your answer.

Consider a equivalence rel^n (a,b), (b,c), (c,d) ∈R then by transitivity (a,c)∈R, (b,d)∈R
(a,c), (c,d) ∈R then (a,d)∈R

Consider nice equi rel^n (a,b) (b,c), (c,d) ∈R then (a,d)∈R
(a,d),(d,c),(c,c) ∈R then (a,c)∈R // (d,c) because of symmetricity
(c,c) because of reflexivity
(b,c),(c,a),(a,d) ∈R then (b,d)∈R

By these, transivity ⟷ Nice transitivity

∴ # nice equi rel^n = # equi rel^n = $B_n = \sum_{k=0}^{n-1} \binom{n-1}{k} B_{n-k-1}$

10. (1.5 marks) Coin change: Given denominations Rupees 1, 3 and 4, prove that a change for any positive integer x can be given using a minimum number of denominations.

Base Case:
$x=1$, $1\times1+3\times0+4\times0$

$x=1$,	$1\times1+3\times0+4\times0$
$x=2$,	$1\times2+3\times0+4\times0$
$x=3$,	$1\times0+3\times1+4\times0$
$x=4$,	$1\times0+3\times0+4\times1$
$x=5$,	$1\times1+3\times0+4\times1$
$x=6$,	$1\times0+3\times2+4\times0$
$x=7$,	$1\times0+3\times1+4\times1$
$x=8$,	$1\times0+3\times0+4\times2$
$x=9$,	$1\times1+3\times0+4\times2$
$x=10$,	$1\times0+3\times2+4\times1$
$x=11$,	$1\times0+3\times1+4\times2$
$x=12$,	$1\times0+3\times0+4\times3$
$x=13$,	$1\times1+3\times0+4\times3$
$x=14$,	$1\times0+3\times2+4\times2$
$x=15$,	$1\times0+3\times1+4\times3$

Indn Hypothesis:
Assume ∀k≥6, ∃a change for k using Rs 1, 3, 4 with a min # of denominations

Indn Step: ∃ a change for (k+1) using Rs 1, 3, 4.

Case i if ∃ atleast one 3 then one 4
$x+1 \Rightarrow x-3+4$
$=x+1$

Case ii if ∃ atleast one 4 and one 1 then it can be replaced with 2 3's
$x+1 \Rightarrow x-5+6$
$=x+1$

1 3's → 1 4's
1 4's → 1 4's + 1 1's
1 4's + 1's → 2 3's
1 1's → 2 1's
2 1's → 1 3's

Case iii If ∃ atleast one 4 then it can be replaced with 1 4's and 1 1's
$x+1 \Rightarrow x-4+4+1$
$=x+1$

~~Change~~ ~~x+3y+4z=6~~

Case iv if ∃ atleast 1's then add one 1's
$x+1 \Rightarrow x+1$

Case v if ∃ atleast 2's then add one 3's
$x+1 \Rightarrow x-2+3=x+1$

11. (1.5 marks) Prove that in any group of n people, there are at least two with equal number of friends. Does the proof guarantee at least three with equal number of friends. Justify.

Case i: If ∃ a person with no. of friends = 0, then range for # friends [0.. n-2] → n-1 values

Case ii: If ∄ a person with # friends = 0, then range for # friends [1.. n-1] → n-1 values

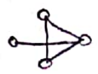
n people (n pigeons), (n-1) pigeon holes. By PHP ∃ hole containing 2 people.

The proof doesn't guarantee for atleast three with equal number of friends. To disprove this claim we need to prove that in any group of n people, ∃ atmost 2 with equal number of friends.

eg

if n = 4, then Case i: 0, 1, 2, 1 →

Case ii: 1, 2, 3, 2 →



∄ exist 3 with equal # friends.

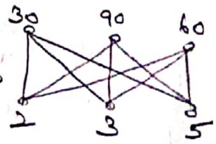
Extra Credit: What is the maximum number of edges (antisymmetric arcs) on a Hasse diagram on n elements. Prove your answer. Is the number tight (exhibit a Hasse diagram meeting the number) ?

Maximum number of edges on n elements = $\left\lfloor \frac{n^2}{4} \right\rfloor$

Induction:-

Base Case: n = 2, $\frac{n^2}{4} = \frac{4}{4} = 1$

n = 6, $\frac{36}{4} = 9$ edges

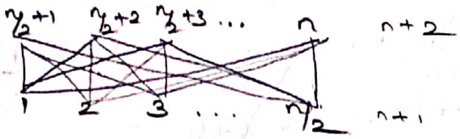


Indn. Hypo: Assume that the max. number of edges on n elements is $\left\lfloor \frac{n^2}{4} \right\rfloor$, ∀ n ≥ 2

Indn. Step:

Case i: If n is even,

Consider a Hasse diagram on n elements



add 2 elements in Hasse diagram on n elements

edges on n+2 Hasse diagram = $(n+1)^{st}$ will be adjacent to $(n/2+1, \dots, n)$ and $(n+2)^{nd}$ will be adjacent to $(1, 2, \dots n/2)$ and (n+1 and n+2 can have edge)

and by hypo $n^2/4$ edges are already present for n vertex Hasse diagram

$= \frac{n^2}{4} + \frac{n}{2} + \frac{n}{2} + 1 = \left(\frac{n}{2}\right)^2 + 2\left(\frac{n}{2}\right) + 1 = \left(\frac{n}{2}+1\right)^2$

$= \frac{(n+2)^2}{4}$

Similarly for odd.

Indian Institute of Information Technology
Design and Manufacturing, Kancheepuram
Chennai 600 127, India
An Autonomous Institute under MHRD, Govt of India
An Institute of National Importance
COM 205T - Discrete Mathematics

End Semester
15-Nov-2019
Duration: 3hr
Marks: 40

Roll No: Name:

0. The book 'The Man who knew infinity' is the biography of

Ramanujam

1 Light Dose

1. (5 marks) Tick all that are true (multiple choice multi correct questions). There is no negative marking, if incorrect. If there is no tick, it is assumed it is false.

1.a)
- A graph G has Eulerian circuit
  - ✓(a) G is connected and $\forall v, d_G(v)$ is even.
  - (b) All vertices in G have even degree.
  - (c) G has a Hamiltonian cycle
  - ✓(d) Line graph of G is also Eulerian.

1.b)
- For the degree sequence (3, 3, 3, 3, 3, 3)
  - ✓(a) There exists an undirected simple bipartite graph.
  - ✓(b) There exists an undirected simple non-bipartite graph.
  - (c) There exists an undirected simple graph with two maximal connected components.
  - ✓(d) There exists a Hamiltonian cycle graph.

1.c)
- In the context of equivalence relations
  - ✓(a) Equivalence classes partition the underlying set irrespective of finite set or infinite set
  - (b) Only for finite sets, the partition is defined.
  - (c) No equivalence relation is a partial order.
  - ✓(d) The index refers to the number of distinct equivalence classes.

1.d)
- Which of the following are uncountable
  - ✓(a) The number of reflexive binary relation on the set of real numbers.
  - (b) The number of functions from $\{a, b, c\}$ to Q.
  - ✓(c) Uncountable union of countable sets is uncountable.
  - ✓(d) The number of computational problems.

1.e)
- $\forall x(P(x) \vee Q(x))$ is equivalent to
  - (a) $\forall x(P(x) \rightarrow Q(x))$
  - ✓(b) $\forall x(\neg P(x) \rightarrow Q(x))$.
  - ✓(c) $\neg\exists x(\neg P(x) \wedge \neg Q(x))$
  - ✓(d) $\forall x(\neg Q(x) \rightarrow P(x))$.

For questions 1.a, 1.c

Correct choice - 0.5 Marks

Wrong choice - 0.25 Marks reduced

For questions 1.b, 1.d, 1.e

Any 2 Correct choices - 0.75 Marks

1 Correct choice - 0.25 Mark

Wrong choice - 0.25 Mark reduced

2. (1 mark) Verify Euler's planarity formula for (i) trees (ii) cycles

(i) $n - e + f = 2$; $n - (n-1) + 1 = 2$ # edges in a tree $= n-1$

(ii) $n - n + 2 = 2$

3. (1 mark) Two sets are related if they both have same cardinality. How many distinct equivalence classes are there. Note: There are three types of cardinalities, namely finite, countably infinite, uncountable. Is this antisymmetric.

Three

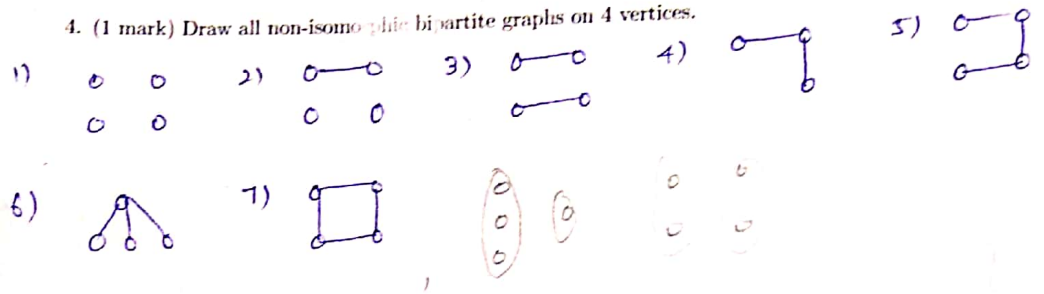
[finite] contains all sets that are finite

[C∞] " C∞

[uncountable] " uncountable.

NO, Not antisymm. ℕ and ℤ are C∞ but not same.

4. (1 mark) Draw all non-isomorphic bipartite graphs on 4 vertices.



5. (1 mark) A = set of C programs that terminate. B = set of $C++$ programs with exactly three input and output statements. With proper justification, compare the cardinalities of A and B.

A : C∞ B : C∞ ⇒ |A| = |B|.

6. (1 mark) Negate and simplify $\forall L \exists n \forall z(|z| \geq n \wedge \exists u \exists v \exists w((z = uvw, |uv| \leq n) \rightarrow \forall i(i \geq 0 \rightarrow uv^iw \in L)))$

$\exists L \forall n \exists z \, (|z| \ngeq n \vee \forall u \forall v \forall w \, ((z = uvw, |uv| \leq n) \wedge \exists i (i \geq 0 \wedge uv^iw \notin L)))$

2 Medium Dose

1. (1.5 marks) Prove or Disprove: (i) $[\forall x P(x) \rightarrow \exists x Q(x)] \rightarrow [\exists x(P(x) \wedge Q(x))]$ (ii) $[\exists x(P(x) \wedge Q(x))] \rightarrow [\forall x P(x) \rightarrow \exists x Q(x)]$

UOD : ℕ

(i) False $P(x) : x \neq x$ $Q(x) : x < 0$

$\forall x \, P(x) = 0$ $\exists x \, Q(x) = 0$ $0 \rightarrow 0$ is True

$\exists x (P(x) \wedge Q(x)) = \text{False}$ True → False is False.

(ii) $\exists x (P(x) \wedge Q(x)) \Rightarrow \exists x \, P(x) \wedge \exists x \, Q(x)$ ← both are true

$P \wedge Q \Rightarrow P \Rightarrow Q$ is a tautology

$\Rightarrow \exists x \, P(x) \rightarrow \exists x \, Q(x)$

Note Both $\exists x \, P(x)$ and $\exists x \, Q(x)$ are True.

Conclusion is Always true

⇒ premise can be any Exp.

$\Rightarrow \forall x \, P(x) \rightarrow \exists x \, Q(x)$

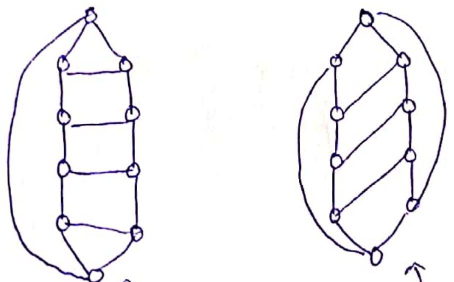
2. (1.5 marks) How many different symmetric binary relations are there on a set of size $n \geq 4$ containing $\{(1,2),(2,3),(3,4)\}$ as a subset. Each symmetric relation must contain this set as a subset.

$\frac{n^2-n}{2}$ symm pairs Subtract 3 $\{(1,2)(2,3)(3,4)\}$

$2^{\frac{n^2-n}{2} - 3} \times 2^n$

↑ Symm pairs. ↑ Diagonal elements

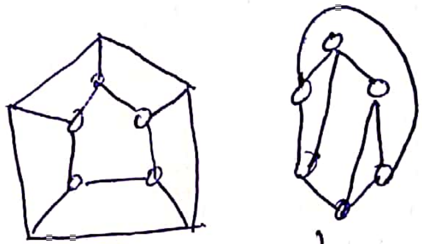
3. (1.5 marks) Draw two different non-isomorphic planar graphs for the degree sequence (3, 3, 3, 3, 3, 3, 3, 3, 3, 3). Intuitively argue that both graphs are non-isomorphic.



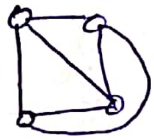
↑ this has Δle

↑ No Δles

Structurally diff Non-iso.



Connected



Dis Connected

4. (1.5 marks) Present a proof without using PIE. The number of onto functions from a set of size n to a set of size 3. Verify your number for $n = 3, n = 4$.

Using PIE: $3^n - {}^3C_1 \cdot 2^n + {}^3C_2 \cdot 1^n = 3^n - 3 \cdot 2^n + 3$

$${}^nC_1 \left({}^{n-1}C_1 + {}^{n-1}C_2 + \cdots + {}^{n-1}C_{n-2} \right)$$

$$+ {}^nC_2 \left({}^{n-2}C_1 + {}^{n-2}C_2 + \cdots + {}^{n-2}C_{n-3} \right)$$

$$+ \cdots + {}^nC_{n-2}$$

n = 3 3! onto fns. = 6 ${}^3C_1 \times {}^2C_1 = 6$

n = 4 $3^4 - 3 \cdot 2^4 + 3 = 36$ ${}^4C_1 ({}^3C_1 + {}^3C_2) + {}^4C_2 ({}^2C_1) = 36$

5. (1.5 marks) Prove or Disprove: Let W_1 and W_2 are well order relations on a finite set. (i) $W_1 \cap W_2$ is a well order (ii) $W_1 \cup W_2$ is a well order

Well order $\Rightarrow$ Total order $\Rightarrow$ poset $\Rightarrow$ R, A, T

(i) $W_1 \cap W_2$:

W_1 ref, W_2 ref $\Rightarrow$ $W_1 \cap W_2$ ref

W_1 Anti, W_2 Anti $\Rightarrow$ $W_1 \cap W_2$ Anti

Trans Tran $\Rightarrow$ $W_1 \cap W_2$ Trans.

$A = \{1,2,3\}$ $\Rightarrow$ $W_1 \cap W_2$ is a finite

$W_1 = \{ (1,1) (2,2) (3,3) (1,2) (2,3) (1,3) \}$

$W_2 = \{ (1,1) (2,2) (3,3) (2,1) (3,2) (3,1) \}$

$W_1 \cap W_2 = \{(1,1) (2,2) (3,3)\}$

Not a total order

$\Rightarrow$ $W_1 \cap W_2$ is not a well order.

(ii) $W_1 \cup W_2$ Antisymm fails for the above ex

$\Rightarrow$ Not a partial order

$\Rightarrow$ Not a well order.

6. (1.5 marks) Show that in any 52 distinct integers, there exist two of them whose sum or else difference is divisible by 100.

$\{0\}$ $\{1, 99\}$ $\{2, 98\}$ ... $\{49, 51\}$ $\{50\}$ 51 boxes

↑ represents remainders.

For a no, x ; perform x % 100 if say, 2, Place in box labeled $\{2, 98\}$.

By PHP ; $\exists$ box contains two pigeons.

If $rem_i = rem_j$ then Diff is div by 100

$rem_i \neq rem_j$ the sum "

7. (1.5 marks) A chess player wants to prepare for a championship match by playing some practice games in 77 days. She wants to play at least one game a day but not more than 132 games altogether. Show that no matter how she schedules the games, there is a period of consecutive days during which she plays exactly 21 games.

a_i : # games played until day i

$a_1 < a_2 < \cdots < a_{77}$ (Atleast one game a day strictly increasing)

~~Add~~ NOTE : $a_1 < a_2 < \cdots < a_{77} \leq 132$ (77 integers - all are distinct)

Consider : $a_1 + 21 < a_2 + 21 \cdots < a_{77} + 21 \leq 153$. (77 integers - all are distinct)

The range of the integer set [1..153]

Range : 153 # integers = 154 (77+77)

Therefore, $a_i + 21 = a_j$ Starting from $a_{i+1}, a_{i+2} \cdots a_j$

$a_j - a_i = 21$ she had played 21 games.

8. (1.5 marks) What is wrong with the following proof; Claim: The number of computational problems is countably infinite. Proof: Assume natural language is the English and the English has well defined alphabet Σ. Note that each problem description is a string in Σ*. Since Σ is finite, Σ* is countably infinite. Therefore, the number of computational problems has got a mapping to the set of natural number.

- The proof Assumes that the problem description is of finite length
- A problem desc can contain an ∞-length substring.
- For example; P_i : print i $i \in [0,1]$

problem desc is infinite.

=) # comp prob = uncountable.

9. (1.5 marks) A = set of equivalence relations $R_i, R_j \in A$ is related if $R_i \subseteq R_j$. (i) Is this relation a partial order. Justify. (ii) What is the least element and greatest element.

1) yes, partial order

2) $R_i \subseteq R_j$; ref , $R_i \subseteq R_j \Rightarrow R_j \not\subseteq R_i$ Antisymm

$R_i \subseteq R_j \subseteq R_k \Rightarrow R_i \subseteq R_k$ trans.

Least element : Equality rel^n {(1,1) (2,2) ... (n,n)}

Greatest element : A X A

10. (1.5 marks) Prove Euler's planarity formula using MI.

We shall prove using Indn on 'f'.

- Case 1 Trees

Base 1 $n-e+f=2$ $f=1$

$n-(n-1)+1=2$

- Case 2 Non Trees.

Base: 2  ; All C_n

$n-n+2$
$=2$

$3-3+2=2$

Hypo Ghs with $f \geq 1$ faces

$n-e+f=2$

Step: Consider a gh with $f \geq 2$ faces.

Since $f \geq 2$, $\exists$ cycle C

Choose an edge in C and delete

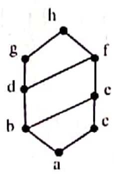
faces decreases by '1'

By Hypo $n-(e-1)+(f-1)=2$

$\Rightarrow n-e+f=2$

hence proved

11. (1.5 marks) For the Hasse Diagram;



- Minimal elements $\{a\}$
- Maximal elements $\{h\}$
- Lower bounds for $\{d, e\}$ $\{a, b\}$
- Greatest lower bound for $\{b, c\}$ $\{a\}$
- Upper bounds for $\{d, e\}$ $\{f, h\}$
- Least upper bound for $\{d, e\}$ $\{f\}$.

12. (1.5 marks) Present an example of algebraic structure for (i) Subgroup but not a monoid (ii) Monoid but not a group (iii) group but not abelian.

(i) $(N-\{0\}, +)$

(ii) $(N+\{0\}, +)$

(iii) $(M_{m\times n}, *)$
↓
Set of all matrices whose determinant $\neq 0$

3 Strong Dose

1. (3 marks) Write FOL using the following notation. Do not use any other notation. $ST(x)$ x is a student, $SE(x)$ x is a semester, $SU(x)$ x is a subject, $L(x,y)$ x likes y, $O(x,y)$ x is offered in y.

- Some students like all subjects offered in every semester.

$$\exists x(ST(x) \wedge \forall z(SE(z) \rightarrow \forall y(SU(y) \wedge O(y,z)) \rightarrow L(x,y)))$$

- There are students who do not like any subject offered in any semester.

$$\exists x(ST(x) \wedge \forall z(SE(z) \rightarrow \forall y(SU(y) \wedge O(y,z)) \rightarrow \sim L(x,y)))$$

- Some students like some subjects offered in some semester.

$$\exists x(ST(x) \wedge \exists z(SE(z) \wedge \exists y(SU(y) \wedge O(y,z) \wedge L(x,y))))$$

- No student likes all subjects offered in a semester.

$$\sim[\exists x(ST(x) \wedge \exists z(SE(z) \wedge \forall y((SU(y) \wedge O(y,z)) \rightarrow L(x,y)))]$$

- In each semester there is a subject which no student likes.

$$\forall z(SE(z) \rightarrow \exists y(SU(y) \wedge O(y,z) \wedge \forall x(ST(x) \rightarrow \sim L(x,y)))))$$

- There is a subject which no student likes irrespective of the semester in which it is offered in.

$$\exists y(SU(y) \wedge \forall z(SE(z) \wedge O(y,z) \rightarrow \forall x(ST(x) \rightarrow \sim L(x,y)))$$

2. (3 marks) Consider two sets; $|A| = m$, $|B| = n$. It is well known that if $m \neq n$ then there are no bijective functions from A to B. In such a case, one can define maximal bijective functions. A subfunction which is maximal and bijective. A set S is maximal with respect to property P if there is no strict superset of S that satisfy P. (i) How many maximal bijective functions are possible when $m \neq n$. Present a clear justification. (ii) Note that functions are graphs. Can you model this counting problem as an appropriate graph theoretic problem. Present a suitable justification.

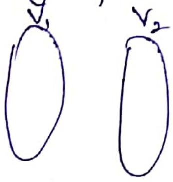
$K_{m,n}$

i) $m < n$, then bijective fn is ~~not possible~~ ${}^{n}C_{m} * m!$

Maximal/maximum matching

$m > n$, then bijective fn is ${}^{m}C_{n} * n!$ (# one-one fn)

ii) How many bipartite graph with V_1, V_2 partitions



$\ni d_i \ \forall$ vertices $i \in V_2 = 1$

$d_i \ \forall$ vertices $i \in V_1 = 1$

d_i for remaining $m-n$ vertices $= 0$

(or)

$|V_1| = m, \ |V_2| = n$

ii) # Maximal / Maximum matching possible in a complete bipartite graph ($K_{m,n}$)

$$= m(m-1)(m-2)\ldots(m-(n-1))$$

$$= \frac{m!}{(m-n)!}$$

$$= \frac{m!}{(m-n)!\,n!} * n!$$

$$= {}^{m}C_{n} * n!$$

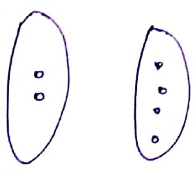
If $m > n$,

for 1st vertex in n, it has m possibilities

2nd vertex in n, it has $(m-1)$ possibilities

⋮

n^{th} vertex in n, has $(m-(n-1))$ possibilities



$\therefore \ {}^{m}C_{n} * n!$

3. (3 marks) Given a set A of size n, how many different ways one can partition A into k parts. For example, $A = \{1,2,3\}$ and $k = 2$ then the different ways are $\{\{1,2\},\{3\}\},\{\{1,3\},\{2\}\},\{\{2,3\},\{1\}\}$. Let this number be P_n. How are B_n (Bell's number) and P_n related. Compute B_1, B_2, B_3 interms of appropriate P_n's.

$$P_{n,k} = k\,P_{n-1,k} + P_{n-1,k-1}$$

$$\underbrace{k=1,\ k=2,\ \dots,\ k=n}_{B_n}$$

P_k^n be number of ways a set of size n can be partitioned into k size partitions.

$$P_1^n = {}^nC_1$$

$$P_2^n = {}^nC_1\,P_1^{n-1} + {}^nC_2\,P_1^{n-2} + {}^nC_3\,P_1^{n-3} \dots$$

$$= \sum_{i=1}^{\lfloor n/2 \rfloor} {}^nC_i\,P_1^{n-i}$$

$$P_3^n = \sum_{i=1}^{\lfloor n/2 \rfloor} {}^nC_i\,P_2^{n-i}$$

$$P_k^n = \sum_{i=1}^{n} {}^nC_i\,P_{k-i}^{n-i} \qquad \therefore\ B_n = \sum_{i=1}^{n} P_i^n$$

$$B_1 = P_1^1 = 1$$

$$B_2 = P_1^2 + P_2^2 = 1+1 = 2$$

$$B_3 = P_1^3 + P_2^3 + P_3^3 = 1+3+1 = 5$$

Strong Dose (or)

3\. # of ways of partitioning A into k parts = $\frac{\text{\# of onto fns}}{k!}$ where $|A| = n$, $|B| = k$

$$(P_{n,k}) = \frac{k^n - \left({}^kC_1 (k-1)^n - {}^kC_2 (k-2)^n + \dots\right)}{k!}$$

$$B_n = \sum_{k=1}^{n} P_{(n,k)}$$

$B_1 = P_{1,1} = 1$

$B_2 = (P_{2,1}) + (P_{2,2}) = 1+1 = 2$

$B_3 = (P_{3,1}) + (P_{3,2}) + (P_{3,3}) = 1+3+1 = 5$

4. (3 marks) Consider an infinite undirected simple graph, graph with infinite vertices. (i) Count (say, finite, countably infinite, uncountable) the number of graphs with a justification (ii) Count the number of graphs with exactly 7 edges.

$V(G) = \infty \leftrightarrow \mathbb{N}$

$E(G) \subseteq V(G) \times V(G)$

$\mathbb{N} . \leftrightarrow \mathbb{N} \times \mathbb{N}$

edges : $C\infty$

powerset (Edges)

= All Graphs .

$P(\mathbb{N})$ = uncountable .

Exactly 7 edges

$\mathbb{N} \times \mathbb{N} \times \cdots \times \mathbb{N}$

$C\infty$.

Extra Credit: (4 marks) How many different partial orders are there on a set of size n. Present a rich combinatorial argument along with good upper bounds/lower bounds, if exact bound is not possible.

Hasse Diag iff Δ^{le} free Ghs.

Counting Δ^{le} free Ghs.

total no. Ghs − Ghs with Δ^{les}

↓

$\triangle$ $2^{{}^nC_2 - 3}$