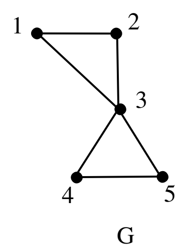
Practice Questions

1. Draw two non-isomorphic graphs for the sequence $(3, 3, 3, 3, 3, 3, 3, 3)$.
   (i) Two K_4's, the graph is disconnected. (ii) Consider a cycle on 8 vertices, C_8. Add the following edges to C_8 to get the desired sequence. $\{\{1, 5\}, \{2, 8\}, \{3, 7\}, \{4, 6\}\}$.

2. Draw a bipartite graph for the sequence $(3, 3, 3, 3, 3, 3, 3, 3)$.
   Consider C_8 augmented with the edge set $\{\{1, 4\}, \{8, 5\}, \{3, 6\}, \{2, 7\}\}$, is an example bipartite graph.

3. What is the largest induced cycle in the above graph.
   C_6. The set $\{1, 4, 3, 6, 7, 8\}$ induces C_6.

4. Consider the Adjacency matrix representation of a simple graph. What does entries A^2 denote? What about A^3, A^k?



$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2 & 3 & 5 & 2 & 2 \\ 3 & 2 & 5 & 2 & 2 \\ 5 & 5 & 4 & 5 & 5 \\ 2 & 2 & 5 & 2 & 3 \\ 2 & 2 & 5 & 3 & 2 \end{bmatrix}$$

   Consider the adjacency matrix representation of the above graph.
   Note that $A^k[i, j]$ represents the number of paths from i to j of length k.
   For example, there exist three paths of length 3 from 1 to 2.
   I.e., $A^3[1, 2] = 3$ and the paths are $(1-2-1-2), (1-3-1-2), (1-2-3-2)$.

5. If G is a simple graph with 15 edges and $\overline{G}$ has 13 edges, how many vertices does G have?
   Total number of edges possible $= \binom{n}{2} = 15 + 13 = 28. \implies n = 8$

6. The maximum number of edges in a simple graph with 10 vertices and 4 components is
   Ans: 21. Three components with K_1's and one component with K_7. Number of edges $= \binom{7}{2} = 21$

7. For which value of k an acyclic graph G with 17 vertices, 8 edges and k components exist?
   Let $e_i, n_i, 1 \le i \le k$ represents the number of edges, and number of vertices in component i, respectively.
   Given $\sum_{i=1}^{k} e_i = 8$
   Since the graph is acyclic it follows that $e_i = n_i - 1, 1 \le i \le k$.
   Therefore, $\sum_{i=1}^{k} (n_i - 1) = 8 \implies \sum_{i=1}^{k} n_i - k = 8$
   Since $\sum_{i=1}^{k} n_i = 17, k = 17 - 8 = 9$

8. Find the minimum number of vertices in a simple graph with 13 edges and having 5 vertices of degree 4 and the rest having degree less than 3.

   Ans: 8. $\sum_{i=1}^{n} d_i = 13 \times 2 = 26$. Number of 4 degree vertices $= 5$. This contributes 20 to the degree. For the rest, we can use vertices of degree less than 3. i.e., there should exist three 2-degree vertices. Therefore, the total number of vertices is at least $5 + 3 = 8$.

9. Does there exist a simple graph with degree sequence $(7, 7, 6, 6, 5, 5, 4, 4, 3, 3, 2, 2, 1, 1)$?

   Ans: Yes.

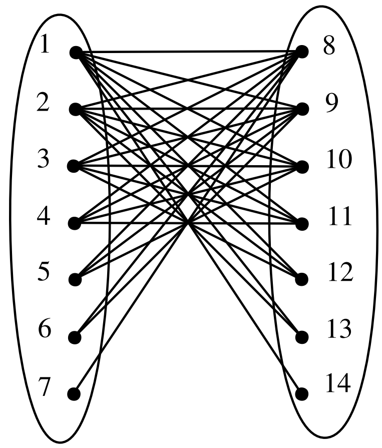


Figure 1: Bipartite graph with the given degree sequence

10. A simple graph G has degree sequence $(3, 3, 2, 1, 1)$. What is the degree sequence of G^c ?

    Ans: $(1, 1, 2, 3, 3)$. For each vertex, the sum of its degrees in G and G^c is $n - 1$. Therefore, $d_{G^c}(v) = (n - 1) - d_G(v)$.

11. If the degree sequence of a simple graph G is $(4, 3, 3, 2, 2)$, what is the degree sequence of G^c ?

    Ans: $(0, 1, 1, 2, 2)$

12. The maximum number of edges in a bipartite graph with n vertices is ...

    Ans: If n is even, then complete bipartite graph $K_{\frac{n}{2},\frac{n}{2}}$ has maximum edges, which equals $\frac{n^2}{4}$. If n is odd, then $K_{\frac{n-1}{2},\frac{n+1}{2}}$ has maximum edges which is $\frac{n^2-1}{4}$.

13. What is the chromatic number of Peterson graph?

    Ans: 3. In the given below graph, 1, 2, 3 represents three different colors.

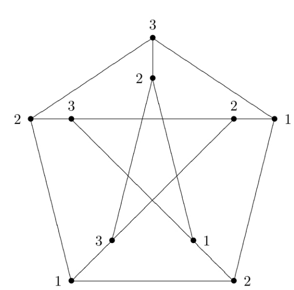


Figure 2: Coloring of a Peterson graph

14. `Claim:` If G contains K_4 as a subgraph, then G is at least 4-colorable. Is this true? Are there 4-colorable graphs without K_4. Are there triangle free graphs with chromatic number 4?
Above claim is true. In the 4-colorable graph shown below, G_1 has no K_3 and G_2 has no K_4 as a subgraph.

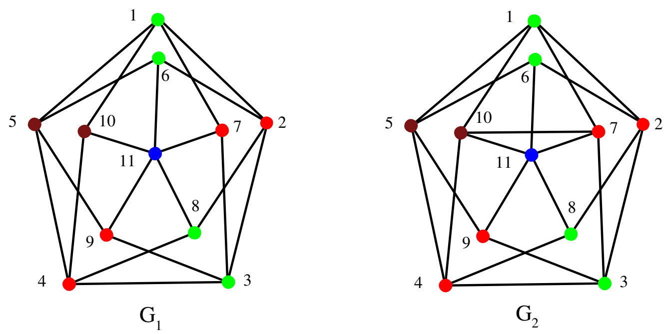


Figure 3: 4-colorable Graphs