Assignment 2 - Solutions

1. Prove or Disprove:
   (a) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists x P(x) \wedge \exists x Q(x)$
   Proof: EI, $P(a) \wedge Q(a)$, for some a
   EG, $\exists x P(x) \wedge \exists x Q(x)$

   (b) $\exists x P(x) \wedge \exists x Q(x) \rightarrow \exists x(P(x) \wedge Q(x))$
   False: UOD: set of natural numbers $P(x)$: x is prime, $Q(x) : x$ is composite. Premise says, there exists a prime number and there exists a composite number, which is true. The conclusion says, there exists a number which is both prime and composite, is false.

2. Check the validity of the following implication. Prove without using truth table.
   $[(P \rightarrow Q) \vee (R \rightarrow S)] \rightarrow [(P \vee R) \rightarrow (Q \vee S)]$
   False: $P : 2+3=6$, $Q : 2+3=-5$, $R : 1+1=2$, $S : 1+1=-2$.

3. Write FOL: We learn something from everyone who passes through our lives. Some lessons are painful, some are painless, but all are priceless. No one who does good work will ever come to a bad end, either here or in the world to come. I am thankful for all of those who said NO to me. It is because of them I am doing it myself.

   $\forall x(person(x) \rightarrow \forall y((person(y) \wedge passthrough(y,x)) \rightarrow \exists z(thing(z) \wedge learn(x,y,z)))) \wedge$
   $\exists x(lesson(x) \wedge painful(x)) \wedge \exists x(lesson(x) \wedge painless(x)) \wedge \forall x(lesson(x) \rightarrow priceless(x)) \wedge$
   $(\neg \exists x(person(x) \wedge doesgoodwork(x) \wedge (badendnow(x) \oplus badendlater(x)))) \wedge \forall x((person(x) \wedge saidno(x)) \rightarrow$
   $thank(me,x) \wedge doing(me,me))$

4. There exists a IIIT where many students are studying. There is a IIIT with no students. Therefore, there are two IIITs such that a student is part of one IIIT whereas he is not part of the other. UOD: Set of students and IIITs. PREDICATES: $STUD(x)$: x is a student. $IIIT(x)$: x is a IIIT. $STUDY(x,y)$: x is studying in y. Do NOT use any other predicates.
   $[\exists x(IIIT(x) \wedge \exists y(stud(y) \wedge study(y,x))) \wedge$
   $\exists x(IIIT(x) \wedge \neg(\exists y(stud(y) \wedge study(y,x)))] \rightarrow$
   $\exists x \exists y(IIIT(x) \wedge IIIT(y) \wedge \exists z(stud(z) \wedge (study(z,x) \oplus study(z,y))))$

5. Consider the academic timetable at IIITDM. UOD: Set of students,courses and time slots. PREDICATES: $STUD(x)$: x is a student. $ELECOURSE(x)$: x is an elective course. $COURSE(x)$: x is a course. $TIMESLOT(x)$: x is a time slot. $TAKEN(x,y)$: x has taken course y. $DAY(x,y)$: (course) x is offered on (day) y. $COURSE-OFFERED-SLOT(x,y)$: x is offered in time slot y. Do NOT use any other predicates.
   Write the FOL for the following.
   Each student has taken at least two elective courses.

   $\forall x(stud(x) \rightarrow \exists y \exists z(y \neq z \wedge elecourse(y) \wedge elecourse(z) \wedge taken(x,y) \wedge taken(x,z)))$

   There exists a student who has courses in all time slots. (there exists a student who has taken at least one course in each time slot)

   $\exists x(stud(x) \wedge \forall y(timeslot(y) \rightarrow \exists z(course(z) \wedge courseofferedslot(z,y) \wedge taken(x,z))))$

There is a student who has not taken a course on any of the time slots on Wednesday.

$\exists x(stud(x) \wedge \neg(\exists y(timeslot(y) \wedge \exists z(course(z) \wedge courseofferedslot(z, y) \wedge day(y, wednesday) \wedge taken(x, z))))$

6. Check the validity.
   a) $\exists x(P(x) \wedge Q(x)) \leftrightarrow \exists x P(x) \rightarrow \exists x Q(x)$
   `Proof of Necessary condn:` EI; $P(a) \wedge Q(a)$, since $(P \wedge Q) \rightarrow (P \rightarrow Q)$ is a tautology, we get $P(a) \rightarrow Q(a)$. On EG, $\exists x P(x) \rightarrow \exists x Q(x)$.
   `Converse is false:` $P(x) : x \neq x$, $Q(x) = x + 2 = 5$

   (b)$\forall x(P(x) \wedge Q(x)) \leftrightarrow \forall x P(x) \rightarrow \forall x Q(x)$

   `Proof of Necessary condn:` UI; $P(a) \wedge Q(a)$, since $(P \wedge Q) \rightarrow (P \rightarrow Q)$ is a tautology, we get $P(a) \rightarrow Q(a)$. On UG, $\forall x P(x) \rightarrow \forall x Q(x)$.
   `Converse is false:` $P(x) : x \neq x$, $Q(x) = x + 2 = 5$

   (c) $\forall x(P(x) \wedge Q(x)) \leftrightarrow \exists x P(x) \rightarrow \exists x Q(x)$.
   Same as (a) as (c) implies (a).
   (d) $\forall x(P(x) \rightarrow Q(x)) \leftrightarrow \exists x(P(x) \rightarrow Q(x))$
   Necessary is trivial as $\forall x$ implies $\exists x$.
   Converse: there exists a number x such that if x is div by 2 then x is div by 6 (this is true), whereas for every number x such that if x is div by 2 then x is div by

7. Prove or disprove: No mathematicians are ignorant. All ignorant people are religious. Therefore, some religious people are not mathematicians.
   False. Venn diagram in which ignorant people are empty and religious people are strict subset of mathematicians. The picture says, all religious people are mathematicians.

8. Prove or disprove: Every living thing is a plant or an animal. Ram's deer is alive and it is not a plant. All animals have legs. Therefore, Ram's deer has legs.
   $[\forall x(l(x) \rightarrow p(x) \vee a(x)) \wedge \forall x(D(x) \rightarrow (alive(x) \wedge \neg p(x))) \wedge \forall x(a(x) \rightarrow leg(x))] \rightarrow \forall x(D(x) \rightarrow leg(x))$